

Solve for $\ddot{\phi}$:

$$g\theta + (-g\phi - l\ddot{\phi} - \dot{x}) + l\ddot{\phi} + \dot{x} - g(\theta - \phi) = 0$$
$$g(\theta - \phi) - g(\theta - \phi) = 0$$
$$0 = 0$$

hmm...

Okay, as on p. 75, let's eliminate ϕ :

$$x_m = x + l\sin\phi + d\sin\theta$$
$$\approx x + l\phi + d\theta$$

$$\phi = \frac{x_m - x - d\theta}{l} \quad \rightarrow \quad \ddot{\phi} = \frac{\ddot{x}_m - \ddot{x} - d\ddot{\theta}}{l}$$

For the 1st EoM:

Thus,

$$g \left(\frac{x_m - x - d\theta}{l} \right) + l \left(\frac{\ddot{x}_m - \ddot{x} - d\ddot{\theta}}{l} \right) + d\ddot{\theta} + \dot{x} = 0$$
$$\frac{g}{l} (x_m - x - d\theta) + \ddot{x}_m = 0$$

Fourier transform:

$$\frac{g}{l} (\tilde{x}_m - \tilde{x} - d\tilde{\theta}) - \omega^2 \tilde{x}_m = 0$$

$$\rightarrow (\omega_0^2 - \omega^2) \tilde{x}_m = \omega^2 (\tilde{x} + d\tilde{\theta})$$

For the 2nd EoM:

$$g\theta + d\ddot{\theta} + l \left(\frac{\ddot{x}_m - \ddot{x} - d\ddot{\theta}}{l} \right) + \ddot{x} + \frac{I}{md} \ddot{\theta} = 0$$

$$g\theta + \ddot{x}_m + \frac{I}{md} \ddot{\theta} = 0$$

Fourier transform:

$$\rightarrow g\tilde{\theta} - \omega^2 \tilde{x}_m - \frac{I}{md} \omega^2 \tilde{\theta} = 0$$

$$\tilde{\theta} = \omega^2 \tilde{x}_m$$