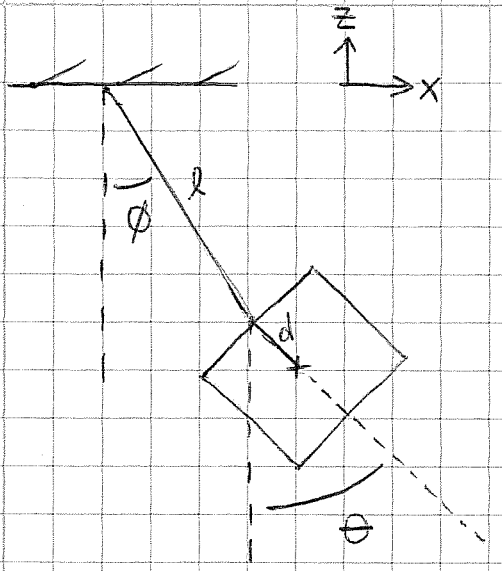
$$\frac{\partial \mathcal{L}}{\partial x_i} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_i} = 0 \qquad \mathcal{L} = T - U$$



$$x = \ell \sin\phi + d\sin\theta$$
$$z = -\ell\cos\phi - d\cos\theta$$

$$\dot{x} = \ell\dot{\phi}\cos\phi + d\dot{\theta}\cos\theta$$
$$\dot{z} = \ell\dot{\phi}\sin\phi + d\dot{\theta}\sin\theta$$

$$I = I_{CM} + md^2$$

$$\begin{aligned} T &= \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}m\dot{z}^2 + \tfrac{1}{2}I\dot{\theta}^2 \\ &= \tfrac{1}{2}m\left(\ell^2\dot{\phi}^2\cos^2\phi + d^2\dot{\theta}^2\cos^2\theta + 2\ell\dot{\phi}d\dot{\theta}\cos\phi\cos\theta\right) \\ &\quad + \tfrac{1}{2}m\left(\ell^2\dot{\phi}^2\sin^2\phi + d^2\dot{\theta}^2\sin^2\theta + 2\ell\dot{\phi}d\dot{\theta}\sin\phi\sin\theta\right) \\ &\quad + \tfrac{1}{2}I_{CM}\dot{\theta}^2 + \tfrac{1}{2}md^2\dot{\theta}^2 \\ &= \tfrac{1}{2}m\ell^2\dot{\phi}^2 + \tfrac{1}{2}md^2\dot{\theta}^2 + m\ell d\dot{\phi}\dot{\theta}\left(\cos\phi\cos\theta + \sin\phi\sin\theta\right) \\ &\quad + \tfrac{1}{2}I_{CM}\dot{\theta}^2 + \tfrac{1}{2}md^2\dot{\theta}^2 \\ &= \tfrac{1}{2}m\ell^2\dot{\phi}^2 + md^2\dot{\theta}^2 + m\ell d\dot{\phi}\dot{\theta}\cos(\phi-\theta) + \tfrac{1}{2}I_{CM}\dot{\theta}^2 \end{aligned}$$

$$\begin{aligned} U &= mgz \\ &= -mg\left(\ell\cos\phi + d\cos\theta\right) \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = m\ell^2\dot{\phi} + m\ell d\dot{\theta}\cos(\phi-\theta)$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = m\ell^2\ddot{\phi} + m\ell d\ddot{\theta}\cos(\phi-\theta) - m\ell d\dot{\theta}(\dot{\phi}-\dot{\theta})\sin(\phi-\theta)$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m\ell d\dot{\phi}\dot{\theta}\sin(\phi-\theta)$$

$$\Rightarrow -m\ell d\dot{\phi}\dot{\theta}\sin(\phi-\theta) - m\ell^2\ddot{\phi} - m\ell d\ddot{\theta}\cos(\phi-\theta) + m\ell d\dot{\theta}(\dot{\phi}-\dot{\theta})\sin(\phi-\theta)$$
$$\Rightarrow -m\ell^2\ddot{\phi} - m\ell d\ddot{\theta}\cos(\phi-\theta) - m\ell d\dot{\theta}^2\sin(\phi-\theta) = 0$$