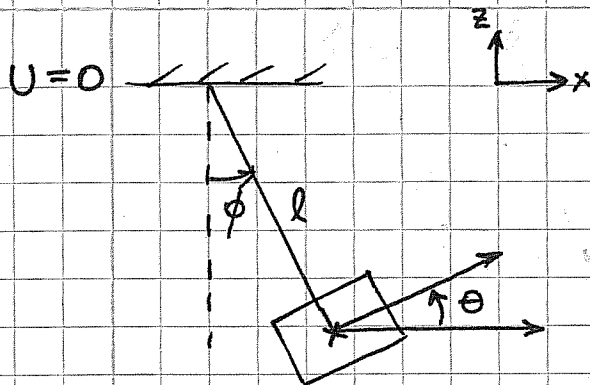


$$\frac{\partial \mathcal{L}}{\partial x_i} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}_i} = 0$$

Mon. Nov. 17, 2014



$$\begin{aligned} T &= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{z}^2 + \frac{1}{2} I \dot{\theta}^2 \\ &= \frac{1}{2} m (l^2 \dot{\phi}^2 \cos^2 \phi + l^2 \dot{\phi}^2 \sin^2 \phi) + \frac{1}{2} I \dot{\theta}^2 \\ &= \frac{1}{2} m l^2 \dot{\phi}^2 + \frac{1}{2} I \dot{\theta}^2 \end{aligned}$$

$$\begin{aligned} U &= mgz \\ &= -mgl \cos \phi \end{aligned}$$

$$\begin{aligned} x &= l \sin \phi & \rightarrow & \dot{x} = l \dot{\phi} \cos \phi \\ z &= -l \cos \phi & \rightarrow & \dot{z} = l \dot{\phi} \sin \phi \end{aligned}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \phi} &= \frac{\partial}{\partial \phi} \left(\frac{1}{2} m l^2 \dot{\phi}^2 + \frac{1}{2} I \dot{\theta}^2 + mgl \cos \phi \right) \\ &= -mgl \sin \phi \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = m l^2 \dot{\phi}$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = m l^2 \ddot{\phi}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = I \dot{\theta}$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = I \ddot{\theta}$$

Lagrange equations of motion

$$-mgl \sin \phi - m l^2 \ddot{\phi} = 0$$

$$I \ddot{\theta} = 0$$

$\ddot{\phi} = -\frac{g}{l} \sin \phi$
$\ddot{\theta} = 0$