$$\frac{\partial \mathcal{L}}{\partial x_i} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_i} = 0$$

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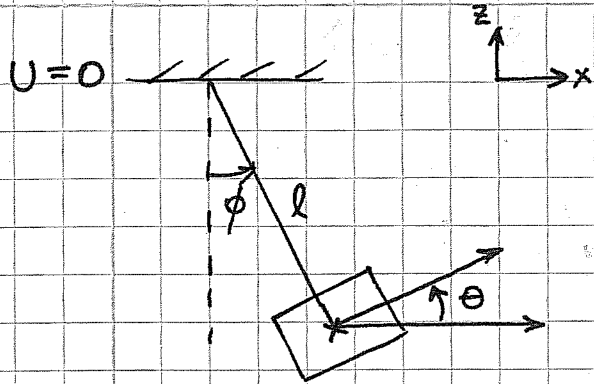

$$T = \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}m\dot{z}^2 + \tfrac{1}{2}I\dot{\theta}^2$$
$$= \tfrac{1}{2}m\left(l^2\dot{\phi}^2\cos^2\phi + l^2\dot{\phi}^2\sin^2\phi\right) + \tfrac{1}{2}I\dot{\theta}^2$$
$$= \tfrac{1}{2}ml^2\dot{\phi}^2 + \tfrac{1}{2}I\dot{\theta}^2$$

$$U = mgz$$
$$= -mgl\cos\phi$$

$$x = l\sin\phi \rightarrow \dot{x} = l\dot{\phi}\cos\phi$$
$$z = -l\cos\phi \rightarrow \dot{z} = l\dot{\phi}\sin\phi$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{\partial}{\partial \phi}\left(\tfrac{1}{2}ml^2\dot{\phi}^2 + \tfrac{1}{2}I\dot{\theta}^2 + mgl\cos\phi\right)$$
$$= -mgl\sin\phi$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = ml^2\dot{\phi} \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = ml^2\ddot{\phi}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0 \qquad \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = I\dot{\theta} \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = I\ddot{\theta}$$

Lagrange equations of motion

$$-mgl\sin\phi - ml^2\ddot{\phi} = 0$$

$$I\ddot{\theta} = 0$$

$$\boxed{\ddot{\phi} = -\frac{g}{l}\sin\phi \qquad \ddot{\theta} = 0}$$