Gyro Displacement Noise Analysis

Zach Korth

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1 Introduction

There has been much waffling about how displacement noise in the gyro cavity mirrors affects the gyro signal. This document is meant to:

- provide a brief review of our waffling, and
- elucidate our current understanding.

2 A (very) brief recap

Initially, we thought that as displacement noise in the cavity mirrors should be common to both directions, we should not see its effect in the gyro signal.

Soon, we realized that the calibration from Hz to rad/s goes as the area of the cavity over the perimeter, and therefore that our calibration would be directly affected by common-mode changes in the cavity length.

Not long after, we realized that the above calibration error would result in a modulation of any existing "DC" rotation signal, such as that from the earth's rotation.

Some time later, we remembered that we are locking one direction to an adjacent axial mode, meaning that we always have a "DC" beat signal at 95 MHz. The expected optical frequency shift from the earth's rotation is $\sim$30 Hz, so this 95-MHz signal is hella bigger.

This is the extent to which we have understood the displacement noise coupling, *until now ...*

3 The current picture

I will begin by describing the mechanism for the FSR modulation, and then I will explain an additional phase-noise coupling.

3.1 FSR modulation

With no rotation, the optical eigenfrequencies in the two directions are:

$$\begin{aligned} \nu_1 &= n\frac{c}{S} \\ \nu_2 &= (n+1)\,\frac{c}{S}, \end{aligned}$$

where n is the axial mode number in the non-upshifted direction and S is the perimeter of the cavity. The beat frequency at the output is thus

$$\nu_2 - \nu_1 = \frac{c}{S},$$

which is precisely the FSR of the cavity.

If there is some perturbation ΔS to the cavity perimeter, the eigenfrequencies become

$$\begin{aligned} \nu_1 &= n\frac{c}{S+\Delta S} \\ \nu_2 &= (n+1)\,\frac{c}{S+\Delta S}, \end{aligned}$$

giving a new beat frequency

$$\nu_2 - \nu_1 = \frac{c}{S+\Delta S}.$$

Since we still believe the FSR to be c/S, we will measure an apparent signal

$$\nu_{sig} = \frac{c}{S+\Delta S} - \frac{c}{S} = -\frac{c\Delta S}{S^2 + S\Delta S}, \tag{1}$$

which corresponds to a rotation signal

$$\Omega_{sig} = \frac{\lambda S}{4A}\nu_{sig} = -\frac{4\lambda c\Delta S}{S^3 + S^2\Delta S}, \tag{2}$$

where $A = (\frac{S}{4})^2$ is the cavity area[1].

Equation 2 encapsulates how displacement noise in the gyro cavity results in a spurious rotation signal via modulation of the FSR.

3.2 Phase modulation

The gyro cavity is a ring cavity, which means that the phase of a circulating beam is only constrained at the input mirror, since this is where the interference used for locking takes place[2]. As a consequence, displacement noise in the cavity mirrors causes the phase at the output mirror to vary. Depending on the relationships between the cavity mirrors' motion, this phase wander can be common-mode or differential between the two counter-propagating beams.

Differential phase drift between the two beams causes phase modulation of the optical beat signal produced at the output beamsplitter. Since we are measuring the frequency of this beat signal using a phase-lock loop (which is phase-sensitive), we will see frequency noise in the gyro signal.

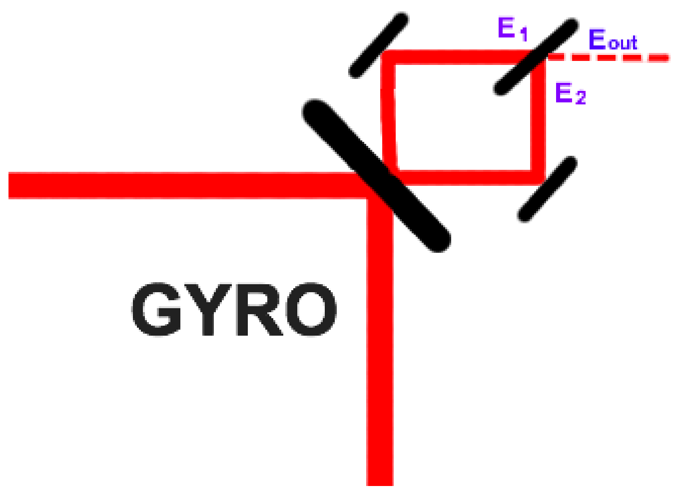

Figure 1: Definition of fields for analysis below. All fields are defined just outside the beamsplitter.

[1]There are of course higher-order corrections due to the change in cavity area, but we have bigger fish to fry.

[2]This does not happen with a linear cavity because in that case the phase displacement at the end mirror must be exactly half what it is for a full roundtrip.

We have known that this effect will take place in the steering mirrors and beamsplitter used in the so-called "transmitted Mach-Zehnder", but we must also contend with it in the gyro cavity itself. This is a much bigger deal because stabilizing a cavity of that size is far more difficult[3].

Figure 1 shows a diagram of the output corner of the gyro. Let the fields $\vec{E_1}$ and $\vec{E_2}$ be the beams reaching the beamsplitter from the (cavity) CCW and CW directions, respectively, and let $\vec{E}_{out}$ be the beat field incident on the PLL photodiode. The beamsplitter is assumed to be ideal, with $r, t = \frac{1}{\sqrt{2}}$. Define the input fields immediately before their interaction with the beamsplitter,

$$\begin{aligned} \vec{E_1} &= Ee^{i\omega_1 t} \\ \vec{E_2} &= Ee^{i[\omega_2 t + \phi(x)]}, \end{aligned}$$

where E is the amplitude of the light emerging from the cavity (equal in both directions), and $\phi(x)$ is a phase shift between the two beams at the beamsplitter, dependent on the differential optical path length seen by the wavefronts in either direction. Differential motion in the curved cavity mirrors or in the transmission optics directly affects ϕ.

Up to an arbitrary overall phase shift, we can write

$$\vec{E}_{out} = \frac{1}{\sqrt{2}}\left(\vec{E_2} + i\vec{E_1}\right) \tag{3}$$

so that

$$P_{out} \equiv |\vec{E}_{out}|^2 = P\left[1 - \sin\left((\omega_1 - \omega_2)t - \phi(x)\right)\right], \tag{4}$$

with $P \equiv E^2$. Thus, the optical beat signal is a sinusoid with $\omega_{out} = \omega_1 - \omega_2$, phase modulated by displacement noise. We can calibrate from displacement noise into frequency noise in the following way. A displacement Δx causes a phase shift

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x.$$

The PLL will act to match the phase of the VCO with that of the beat signal. If there is a phase advance, the loop will increase the VCO frequency to catch

[3]We could have, for example, designed some monolithic setup for the transmission readout optics to reduce noise therein, but this is impractical for the large gyro cavity.

up, then reduce it to keep it from getting ahead. The frequency noise, in the Fourier domain, is

$$\widetilde{\Delta f} = 2\pi f \widetilde{\Delta\phi} = \frac{4\pi^2 f}{\lambda}\widetilde{\Delta x},$$

so the angular velocity noise is

$$\widetilde{\Delta\Omega} = \frac{\lambda S}{4A}\widetilde{\Delta f} = \frac{16\pi^2 f}{S}\widetilde{\Delta x}. \tag{5}$$

4 Measurements

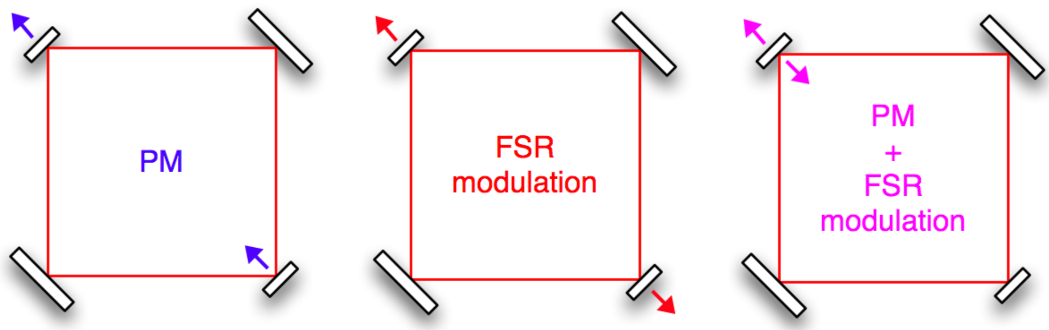

Figure 2: Noise coupling expected from different modes of mirror motion.

Figure 2 shows three examples of cavity mirror motion. On the left is the case when both curved mirrors move in unison. In this case, the cavity length and area are not changed to first order, but there is phase modulation of the output beat field. At center, both mirrors move in opposite directions. In this case, the cavity perimeter and area change but there is no additional phase shift incurred at the beamsplitter. Hence, there is frequency noise from FSR modulation but no phase noise[4]. Finally, there are superpositions of these modes, such as when one mirror moves alone, causing phase noise and FSR modulation.

Given this, it seems appropriate to apply the noise from the MZ measurement through the phase noise pathway, while applying the single-arm

[4] Motion of the input and output coupling mirrors also falls under this category.

measurement noise through the FSR modulation pathway. This is not exact, but it serves as a reasonable consideration of how both sources affect the system. Figure 3 shows these two contributions and their sum alongside the gyro noise spectrum.

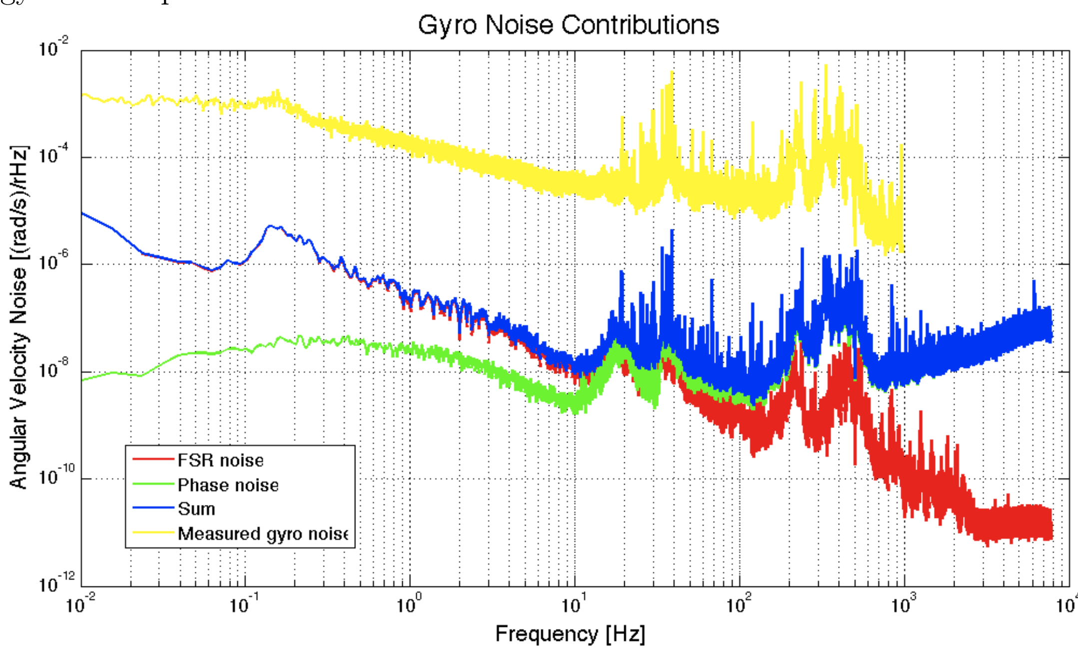

Figure 3: Estimated FSR modulation (red) and phase noise (green) contributions to gyro noise spectrum. Sum is in blue, and the measured gyro noise is in yellow.

From the figure, it is fairly clear that the total gyro noise bears the shape of the two noise sources combined in this way. The only problem is that it is nearly four orders of magnitude higher, and there's no obvious reason why.

There is one other piece of evidence that leads me to believe that at least the phase noise contribution is treated correctly. Figure 4 shows a gyro noise spectrum that Alastair took without modulation to the Tektronix VCO driving the AOM. In this situation, the CW beam offset from the CCW beam by a fixed frequency. Thus, there is no gyro signal, and there is no modulation of the beat signal due to changes in cavity length; that is, we expect the FSR noise to be absent. The phase noise should still be as present as ever, though, as there is no change to this mechanism.

A look at the plot reveals some broadband noise floor, above which the trace only rises between about 350-600 Hz. Noting that the calibration in

Fig. 4 is *overestimated* by 2π, this excursion could be explained by phase noise as estimated in Fig. 3[5].

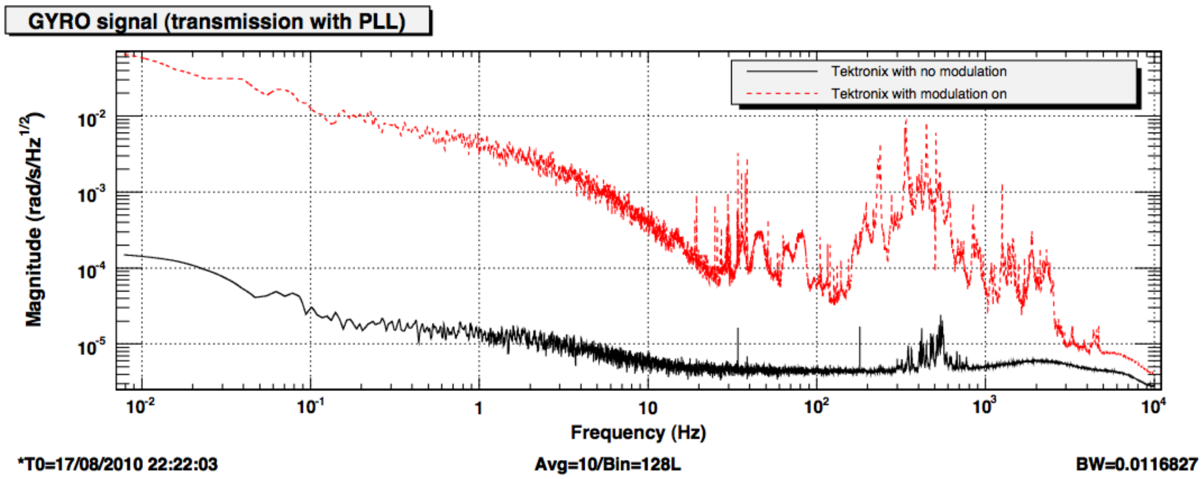

Figure 4: A gyro spectrum with AOM modulation off (black). Calibration is off by 2π and traces should be *lowered* mentally.

If this is not some coincidence, then it suggests that the noise sources discussed above *do* exist at these calculated levels, and are amplified up somehow by our feedback loops. Indeed, the noise levels above are calculated assuming ideal loops working exactly as designed. Subject to calibration error, they constitute fundamental limits to the gyro's sensitivity given the measured displacement noise level.

Of course, there exist several means by which we can improve the situation. Simply putting windows on the box may have already done so, though we have not yet gotten a new spectrum. Going to vacuum will also help a great deal to reduce air-related noise. Finally, we can move to the scheme in which we lock the cavity to stable light via PZT, though, in light of the phase noise mentioned above, this will have to be done cleverly so as to avoid adding in extra noise. It may not even be possible.

While all these things can be done to reduce our fundamental noise threshold, we still need to figure out why the gyro signal is so much noisier than it should be. How does the noise get amplified by the loops?

[5]Note that it is the green curve that matters here, not the blue one. The noise from 8-40 Hz in the green curve is roughly a factor of two lower than that in the blue, and would be swallowed by the broadband noise in Fig. 4.