Shot Noise Limit for Gyro Sensitivity

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The frequency difference Δf created by an angular velocity ω is given by

$$\Delta f = \left(\frac{4}{\lambda S}\right) \mathbf{A} \cdot \boldsymbol{\omega}, \tag{1}$$

where S is the cavity perimeter and $\mathbf{A}$ is the enclosed area vector. The shot-noise-limited uncertainty in measuring an angular speed ω about an axis perpendicular to the plane of the gyro is then

$$\delta\omega = \left(\frac{\lambda S}{4A}\right) \delta f, \tag{2}$$

where

$$\delta f \approx \frac{\sqrt{2}\Gamma}{(N_{ph}\eta\tau)^{1/2}} \tag{3}$$

is the shot-noise-limited uncertainty in the measurement of Δf[1]. Here, Γ is the FWHM of the cavity,

$$\Gamma = \frac{\Delta\nu_{FSR}}{\mathcal{F}}, \tag{4}$$

N_{ph} is the photon arrival rate,

$$N_{ph} = \frac{P}{E_\gamma} = \frac{P\lambda}{hc}, \tag{5}$$

η is the quantum efficiency of the detector, which for a PDA255 at 1064 nm is $\eta \approx 0.82$, and τ is the integration time.

Putting this all together gives

$$\delta\omega \approx \frac{\Delta\nu_{FSR} S}{\sqrt{8} A \mathcal{F}} \sqrt{\frac{hc\lambda}{P\eta\tau}}. \tag{6}$$

The following is a contour plot of $\delta\omega$ as a function of $\mathcal{F}$ and P. Here, a free spectral range of $\Delta\nu_{FSR} = 160$ MHz is assumed (due to considerations for the AOM), giving $S = 1.875$ m and $A = 0.169$ m^2. For comparison, the earth's rotation rate is $\omega_E = 7.27 \times 10^{-5}$ rad/s, well above the range of this graph.

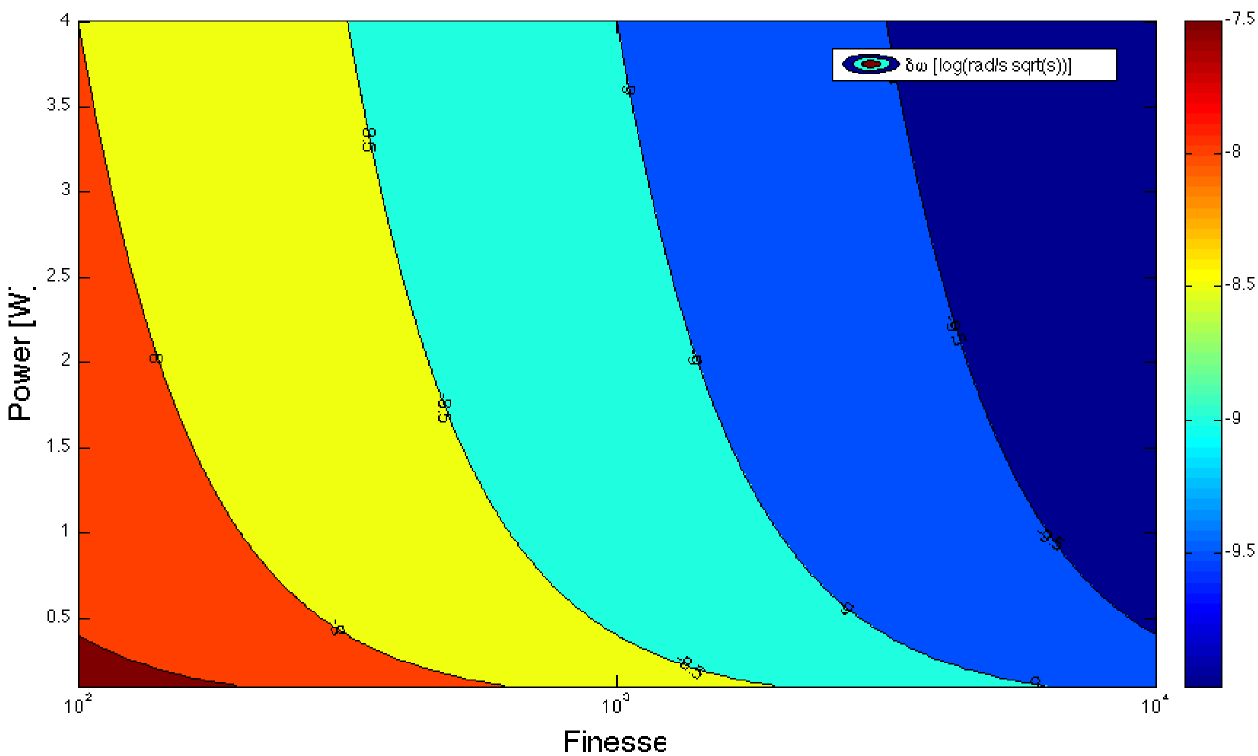

Figure 1: Shot-noise limited sensitivity to angular speed ($\delta\omega$) as a function of finesse ($\mathcal{F}$) and power P. Units are log(rad/s $\cdot \sqrt{\text{s}}$)—that is, a 1-s integration time is assumed.

References

[1] G. A. Sanders, M. G. Prentiss, and S. Ezekiel. Passive ring resonator method for sensitive inertial rotation measurements in geophysics and relativity. *Optics Letters* 6(11):569–571, November 1981.