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17 August 1998

Physics Letters A 245 (1998) 203-208

PHYSICS LETTERS A

Mirror electrostatic suspension for interferometric detectors of gravitational waves

Adalberto Giazotto [1]

INFN Sezione di Pisa, Via Vecchia Livornese 1291, 56010 S. Piero A Grado. Pisa, Italy

Received 11 March 1998; revised manuscript received 13 May 1998; accepted for publication 14 May 1998
Communicated by J.P. Vigier

Abstract

The possibility is evaluated of suspending the mirrors of interferometric detectors of gravitational waves, by means of electrostatic forces, with the purpose of reducing the pendulum thermal noise. An estimate of both damping mechanisms and pendulum frequency is also given. © 1998 Elsevier Science B.V.

1. Introduction

Interferometric gravitational wave (GW) detectors [1] use light to measure the displacements of test masses due to GW; test masses are very high quality mirrors whose intrinsic vibrations, produced by thermal noise (TN), are the main source of displacement noise at low frequency (10–300 Hz). TN occurs because of two main sources of dissipation [2,3]:

(1) The mirror's bulk material, silica or sapphire, has normal modes whose mechanical damping is responsible for TN tails extending to low frequency with a displacement spectral density $\propto 1/\nu^{0.5}$ [4], where ν is the frequency.

(2) Mirrors are suspended with metallic or insulating (silica, sapphire, etc.) wires. Wire connections dissipate energy and give a TN tail having a displacement spectral density $\propto 1/\nu^{0.25}$ [4].

At low frequency (10–100 Hz) mechanism (2) dominates the interferometer's sensitivity; this paper is, hence, meant to give some ideas about the possibility of suspending the interferometer's mirrors by means of electrostatic forces with the purpose of avoiding mirror suspension wires.

Unlike mirror magnetic suspension that requires a magnet-mirror connection, which is expected to spoil the mirror mechanical quality factor Q_M, electrostatic levitation could be performed by depositing on the mirror very thin metal coatings, certainly not more harmful, Q_M-wise, than the 24 $\lambda/4$ layers required for the mirror optical performances.

It is also important to note that a condenser working at constant charge with an electric field of 10^7 V/m is currently being used as a sensor in gravitational bar detectors [5]; in this device no extra noise due to the high electric field has been detected. Furthermore, this field is currently obtainable at room temperature [6].

The bar detectors the condenser's gap is of the order of 30μm to give high displacement sensitivity; this imposes extremely smooth metal surfaces to avoid obscure discharge. In the case of electrostatic levitation this smoothness is not needed anymore because gaps can be several mm.

It is then reasonable to assume that an electrostatic

[1] E-mail: giazotto@pi.infn.it.

PII S0375-9601(98)00411-3

mirror levitation should not give extra noise due to the presence of the high electric field.

The purpose of this work is to propose a suspension method which could satisfy the following requirements:

(1) Allow for the elimination of the mirror suspension wires.

(2) Provide a pendulum of high mechanical quality factor Q_P.

(3) Ensure the mirror to have high Q_M and high frequency internal modes.

2. The method

Let us consider a plane square condenser with area $S = L \times b$ and separation $2D$ as shown in Fig. 1.

This condenser can be topologically deformed to a shape, shown in Fig. 2, which allows the mirror to be attracted *without any wire connection to the mirror itself*. The energy is $E = \frac{1}{2} \times 2Q^2/C$, where $C = \epsilon_0 S/D$ ($\epsilon_0 = 8.89 \times 10^{-12}$ UMKS).

At *constant charge* Q the total attractive force is $F = \delta E/\delta D = Q^2/\epsilon_0 S$, hence the force for each surface S is

$$F = \frac{Q^2}{2\epsilon_0 S}. \tag{1}$$

The charge Q obtained by working at 10^7 V/m [1] and with $S = 10^{-2}$ m^2, is $Q = CV = \epsilon_0 S(V/D) = 8.89 \times 10^{-12} \times 10^{-2} \times 10^7 \approx 10^{-6}$ coulomb.

With this Q value the force is

$$F \approx \frac{Q^2}{2\epsilon_0 S} = \frac{0.5 \times 10^{-12}}{8.89 \times 10^{-12} \times 10^{-2}} \approx 5\,\mathrm{N}$$

for each surface S. *This force is not at all negligible.*

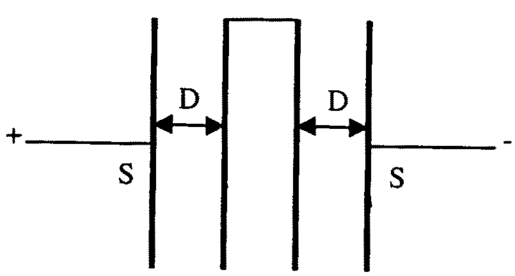

Fig. 1. Two condensers in series; the topological deformation of this diagram can lead to a wireless electrostatic levitation of a test mass.

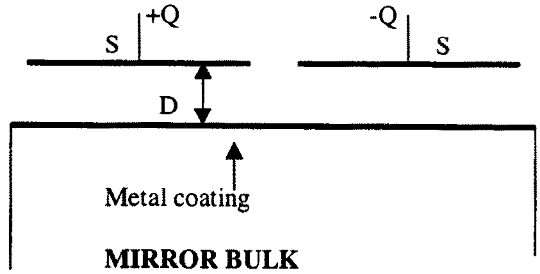

Fig. 2. The topologically deformed configuration originating from the diagram shown in Fig. 1. With this configuration the mirror can be levitated without any wire connection.

3. Mirror shape

As an example let us consider a silica monoblock with size $0.3 \times 0.3 \times 0.1$ m^3, machine carved with a shape as shown in Fig. 3. Fig. 3b shows that the mirror has two surfaces S_1 and S_2 with size $\approx 0.3 \times 0.3$ m^2 whose purpose is twofold:

(1) To allow the mirror to be used as the far mirror, i.e. to reflect beams with a diameter of ≈ 30 cm.

(2) To make very rigid the mirror teeth structure, shown in Fig. 3a, with the purpose of giving very high frequency internal modes.

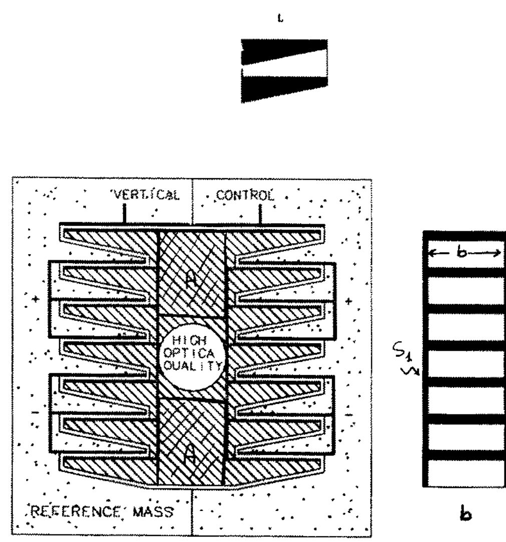

Fig. 3. (a) Mirror and reference mass: both have been carved in two monolithic bulks of optical silica or sapphire. (b) Side view of the mirror: the mirror teeth are all connected by the two optical surfaces, S_1, S_2, and allow a more rigid structure.

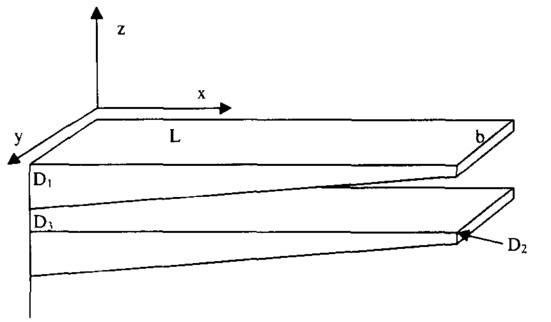

Fig. 4. The mirror teeth structure shown without the two connecting surfaces S_1, S_2.

For this mirror shape, the volume is about 6.5×10^{-3} m^3, which gives for silica (density $\rho = 2.2 \times 10^3$ kg/m^3) a weight $P \approx 14$ kg corresponding to a force of 140 N. This weight can be reduced to ≈ 10 kg by removing material from the zones A (see Fig. 3a).

With reference to Fig. 3, for levitating 100 N, we should need the total sensitive area $20S$; for this case 18 teeth are needed with a shape as shown in Fig. 4, where $L = 10^{-1}$ m, $D_1 = 2 \times 10^{-2}$ m, $D_2 = 10^{-3}$ m, $D_3 = 10^{-2}$ m and width b. The elastic constant of this beam is

$$K(x) = \frac{EbD_1^3}{2x^3[1 + x(D_1 - D_2)/2LD_1]}, \quad (2)$$

where E is the mirror's Young modulus. If $D_2 \ll D_1$

$$K(x) = \frac{EbD_1^3}{2x^3(1 + x/2L)}. \quad (3)$$

The CMS is at $x = L/3$ and the mass is

$$M = \rho D_1 Lb/2, \quad (4)$$

where ρ is the mirror density; the first mode frequency is

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{K(L/3)}{M}} = \frac{D_1}{2\pi L^2}\sqrt{\frac{162E}{7\rho}}. \quad (5)$$

We can evaluate ν_1 for silica ($E = 7 \times 10^{10}$ N/m^2, $r = 2.2 \times 10^3$ kg/m^3) and sapphire ($E = 3.5 \times 10^{11}$ N/m^2, $r = 4 \times 10^3$ kg/m^3),

$$\nu_{1\text{silica}} = 4.5 \times 10^5 D_1 \text{ Hz}, \quad (6)$$

$$\nu_{1\text{sapphire}} = 7.5 \times 10^5 D_1 \text{ Hz}. \quad (7)$$

Hence from Eqs. (6), (7) we expect that every cm of increase of D_1 improves $\nu_{1\text{silica}}$ by 4.5×10^3 Hz and $\nu_{1\text{sapphire}} = 7.5 \times 10^3$ Hz; furthermore, in this calculation, teeth are assumed to be free and not constrained to surfaces S_1 and S_2. These constraints should very much increase the mode frequency. It is interesting to note that the lowest frequency mode of the mirrors to be mounted on the large interferometers such as Virgo and LIGO is $\approx 5 \times 10^3$ Hz.

This evaluation is made to show that the mode frequency of the unconstrained teeth can be very high; it is, anyway, obvious that only a finite element mirror mode analysis can give the right optimization.

4. Mirror stability

This levitation system is stable in the horizontal direction because condensers try to maximize the capacity, while it is unstable in the vertical direction. This means that we should have an *active system*, which could read (interferometrically) the vertical position of the mirror in such a way that a controlled voltage is applied to some of the reference mass condensers. The vertical displacement sensitivity of the reading system should be $\leqslant 10^{-16}$ m/$\sqrt{\text{Hz}}$ at 10 Hz in such a way that, with a vertical to horizontal coupling $\approx 10^{-3}$, the horizontal noise is $\leqslant 10^{-19}$ m/$\sqrt{\text{Hz}}$ at 10 Hz. *This sensitivity seems easily obtainable.*

The mirror is stable in the x–y plane (horizontal plane) but is unstable along the z direction and for rotations around the x and y axes, i.e. θ_x, θ_y. We need a stabilization system reading z, θ_x, θ_y by means of three sensors, each varying the voltage of a dedicated condenser. The stabilization system does not work with constant charge anymore. A possible topology of this system is sketched in Fig. 5.

With reference to Fig. 5, S_1, S_2, S_3 are sensors mea-

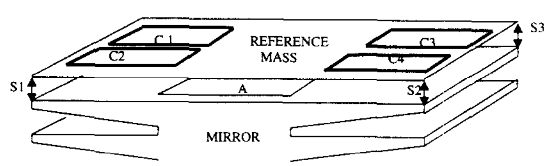

Fig. 5. On the reference mass upper plane (the one above the mirror top) the electrodes are differentiated for creating tilting and stabilizing forces on the mirror. The variable potentials are given to the condensers $C_1 \ldots C_4$, driven by three sensors $S_1 \ldots S_3$.

suring z, θ_x, θ_y, while $C_1 \ldots C_4$ are the metal coatings creating the variable forces driven by sensors $S_1 \ldots S_3$. As an example, the condenser having the connection $(C_1 + C_2, C_3 + C_4)$ is controlling z, (C_1, C_3) and (C_2, C_4) is controlling θ_x, while (C_1, C_2) and (C_3, C_4) is controlling θ_y. A procedure for mirror levitation could be to fill a charge Q on the reference mass condensers to leave the mirror *unlevitated* but with very low weight (10–20 g), and then to use $S_1 \ldots S_3$ for final levitation.

5. Horizontal motion elastic constant (EC)

As a simple example let us consider two charge Q uniform distributions posed on two *insulating planes* as in Fig. 6. By moving plane 2 along the y axis we have the force (see Appendix A)

$$F_y = -y\frac{NQ^2}{Lb^2}\frac{1}{2\pi\epsilon_0}\log\frac{D^2+(b+\Delta)^2}{D^2+\Delta^2}, \quad |y| \ll D, \Delta, \tag{8}$$

where N is the number of condensers with area $L \times b$ The EC is

$$k = \frac{NQ^2}{Lb^2}\frac{1}{2\pi\epsilon_0}\log\frac{D^2+(b+\Delta)^2}{D^2+\Delta^2}, \quad |y| \ll D, \Delta. \tag{9}$$

From Eq. (1) it follows that for levitating a mass m the electrostatic force should be

$$F = \frac{NQ^2}{2\epsilon_0 Lb} = mg, \tag{10}$$

where $g = 9.81$ m s^{-2} is the gravity acceleration and m the mirror mass. From Eqs. (10), (10) we obtain the pendulum horizontal frequency

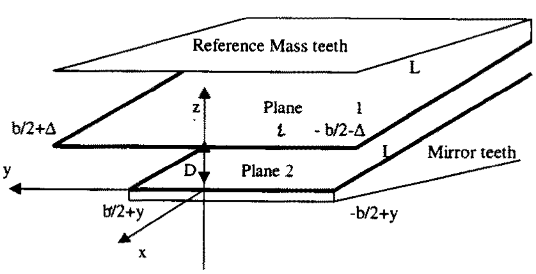

Fig. 6. The reference system used for evaluating the pendulum frequency. The mirror is pendulating along the y axis.

$$\nu_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\pi b}\log\frac{D^2+(b+\Delta)^2}{D^2+\Delta^2}}. \tag{11}$$

Assuming $b = 10^{-1}$ m, $D = 3 \times 10^{-3}$ m, $D = 5 \times 10^{-3}$ m, we obtain $\nu_0 \approx 2.5$ Hz.

6. Mirror damping mechanisms

6.1. Currents generated by mirror-reference mass (RM) relative motion

If the mirror moves with respect to RM the charge Q creates a current i in the metal coatings which dissipate energy by the Joule effect. This dissipation mechanism has also been discussed in Refs. [7,8]. The total energy ϵ of the levitated mirror is

$$\epsilon = \tfrac{1}{2}m\dot{y}^2 + \tfrac{1}{2}ky^2 + \int_0^t i^2 R\,dt, \tag{12}$$

where $i = dQ/dt = Q/b\dot{y}$, m is the mirror mass and the coating's electrical resistance $R = \rho b/L\eta$, where $\rho = 2 \times 10^{-8}$ $\Omega\times$m (for gold) and $\eta = 10^{-6}$ m are the metal coating resistivity and thickness respectively (see Fig. 7). Differentiating ϵ with respect to t we obtain the equation of motion

$$m\dot{y}\ddot{y} + ky\dot{y} + Ni^2R = m\dot{y}\ddot{y} + ky\dot{y} + N\frac{Q^2}{b^2}\dot{y}^2\frac{\rho b}{L\eta}$$
$$= m\dot{y}\ddot{y} + ky\dot{y} + N\frac{10^{-12}}{10^{-2}}\frac{2\times10^{-8}}{10^{-6}}\dot{y}^2 = 0 \tag{13}$$

where N is the total condenser area measured in units of S.

By dividing Eq. (13) by $m\dot{y}$, we obtain the pendulum equation of motion

$$\ddot{y} + \omega^2 y + \frac{N}{m}\frac{10^{-12}}{10^{-2}}\frac{2\times10^{-8}}{10^{-6}}\dot{y} = 0, \tag{14}$$

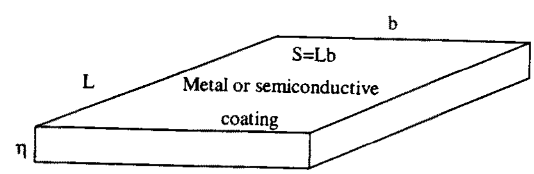

Fig. 7. Geometrical parameters of the thin metallic layers coating mirror and reference mass teeth.

where $\omega_0 = \sqrt{k/m}$ is the pendulum frequency.

Assuming a mirror geometry with $N \approx 33$, $m \approx 14$ Kg, we obtain for the pendulum relaxation time τ,

$$\tau = \frac{mbL\eta}{NQ^2\rho} \simeq 2 \times 10^{11}\,\text{s}. \tag{15}$$

From Eq. (11) it follows that $\omega_0 \approx 15$ rad/s, then the pendulum quality factor, due to this loss mechanism, is $Qp = \omega_0\tau/2 \approx 1.5 \times 10^{12}$, i.e. very large indeed.

The current i can also be expressed as a function of the charge mobility μ and by the tangential (to plane 2 of Fig. 6) electric field E_T in the following way,

$$i = \frac{dQ}{dt} = \frac{Q}{b}(\dot{y}_0 - \dot{y}) = \frac{Q}{b}\mu E_T,$$

where y_0 is the center-of-mass coordinate of the induced charge Q. The resistance is

$$R = \frac{V}{i} = \frac{bE_T}{i} = \frac{b^2}{Q\mu}.$$

The dissipative term becomes

$$Ni^2R = NQ\mu E_T^2.$$

By using *semiconductive* coatings, the charge mobility could be reduced then, making dissipation negligible.

6.2. Electromagnetic waves (EM) emission

The power radiated by EM emission is

$$W = \frac{1}{3\pi}\frac{\mu_0}{c}N^2Q^2\ddot{y}^2, \tag{16}$$

where $\mu_0 = 12.56 \times 10^{-7}$ UMKS and $c = 3 \times 10^8$ m/s.

To evaluate the damping effect due to EM emission let us evaluate the ratio

$$u = \frac{W}{i^2R} = \frac{N^2Q^2\mu_0}{3\pi c}\frac{b^2\eta L}{NQ^2\rho b}\frac{\ddot{y}^2}{\dot{y}^2}. \tag{17}$$

In frequency space, Eq. (17) can be written as

$$u = \frac{\mu_0 bL\eta}{3\pi c\rho}N\omega^2 \approx \frac{10^{-6} \times 10^{-2} \times 10^{-6}}{10 \times 3 \times 10^8 \times 2 \times 10^{-8}}30\omega^2 \approx 10^{-14}\omega^2 \ll 1. \tag{18}$$

Eq. (18) shows that radiation damping in our frequency range is absolutely negligible.

7. Conclusions

Even if the construction of the proposed mirror-reference mass structure seems to be mechanically complex, it is the author's feeling that it is no more complex, TN-wise, than the optimization of a wire suspended mirror system.

It is then evident that only the construction of a prototype could unveil advantages and disadvantages of the proposed solution, with particular reference to the damping mechanism and to the noises produced by high voltage operation on the interferometer's test masses; GW bar detectors give, anyway, positive indication of the smallness of the latter.

Acknowledgement

I am very grateful to C. Bradaschia, M. Cerdonio, L. Gammaitoni, J. Hough, D. Passuello, P. Rapagnani, F. Ricci, A. Viceré and J.Y. Vinet for very useful discussions.

Appendix A

To evaluate the mirror horizontal motion EC, we consider, with reference to Fig. 6, the case of two charges $+Q$ and $-Q$ uniformly spread on the planes 1 and 2 respectively, under the hypothesis that the charges can not migrate, i.e. the planes are insulating.

The force on the y direction is

$$F_y = \frac{dQ}{du}\frac{dQ}{dw}\frac{1}{2\pi\epsilon_0 L} \times \int_{-b/2-\Delta}^{b/2+\Delta} du \int_{-b/2+y}^{b/2+y} dw \frac{u-w}{(u-w)^2 + D^2}. \tag{A.1}$$

The u and w coordinates are the y axis of planes 1 and 2, respectively, $dQ/du = Q/(b+2\Delta) \approx Q/b$ and $dQ/dw = Q/b$. To write Eq. (A.1) we have applied Gauss theorem by subdividing planes 1 and 2 in thin slices with areas $L\,du$ and $L\,dw$, respectively, i.e. we evaluate the y component of the force between two charged parallel (to the x axis) wires having length L, charge $du\,Q/b$ and $dw\,Q/b$, diameter du and dw, and separation $[(u-w)^2 + D^2]^{1/2}$. Since the integral in Eq. (A.1) takes its largest contribution from the region we consider this approximation good enough to evaluate the horizontal motion rigidity.

With easy algebra Eq. (A.1) becomes

$$F_y = \frac{Q^2}{b^2}\frac{1}{4\pi\epsilon_0 L}\int_{-b/2-\Delta}^{b/2+\Delta} \log\frac{(u-b/2-y)^2+D^2}{(u+b/2-y)^2+D^2}\,du. \tag{A.2}$$

The EC along the y axis is obtained by differentiating Eq. (A.2) with respect to y around the point $y = 0$:

$$k = \frac{Q^2}{b^2}\frac{1}{4\pi\epsilon_0 L}\frac{\partial}{\partial y} \times \left(\int_{-b/2-\Delta}^{b/2+\Delta} \log\frac{(u-b/2-y)^2+D^2}{(u+b/2-y)^2+D^2}\,du\right)_{y=0}$$

$$= \frac{Q^2}{b^2}\frac{1}{2\pi\epsilon_0 L}\int_{\Delta^2}^{(b+\Delta)^2} \frac{1}{s+D^2}\,ds.$$

Hence, for N condensers, we obtain the elastic constant

$$k = \frac{NQ^2}{b^2}\frac{1}{2\pi\epsilon_0 L}\log\frac{D^2+(b+\Delta)^2}{D^2+\Delta^2} \tag{A.3}$$

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