July 31, 2014

I have been trying to compare the numerical model with the analytical calculation. The analytical calculation runs faster if the beam radius is made much larger, so I increased it to 1 mm. Furthermore, increasing the length scales by a factor of 2 also sped up the analytical calculation. However, the COMSOL model runs slower, using the same 1-6-6 sandwiching mesh style as usual. I had to resort to a 3-6-6 meshing (finer meshing in the middle), which produced a numerical plot that is clearly unsatisfactory.

Parameters: $L = W = H = 4$ cm, $T = 120$ K, $r_0 = 1$ mm, $\beta = 8.7 \times 10^{-5}$ K^{-1}, $C_p/\rho = 328$ J/kg/K, $\rho = 2331$ kg/m^3, and $\kappa = 615$ W/m/K.

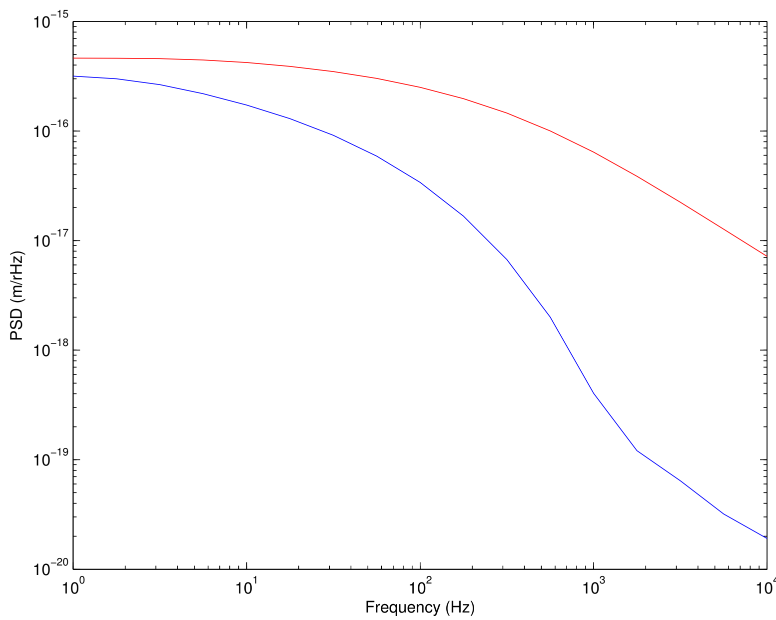

Figure 1: The meshing for the blue numerical curve is not adequate for $r_0 = 1$ mm.

There are some issues concerning how many terms to keep in the analytical Fourier series. Since each component of the Fourier series has a term like $\cos(l\pi x/W)$, our maximum term $l_{\max}$ should satisfy $W/l_{\max} \sim r_0$, since the beam radius is typically the smallest length scale (the other candidate for the smallest length scale is the thermal wavelength $r_{\text{th}} = \sqrt{\kappa/(\omega C_p)}$. In this plot, I went to $l = n = m = 34$.

I ran the simulation again for a uniform extra fine mesh. This meshing actually gives closer results to the analytical solution:

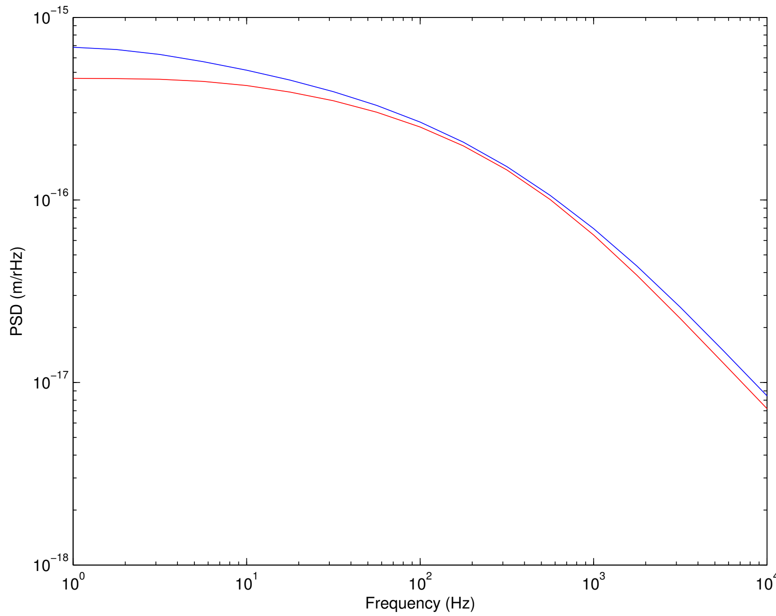

Figure 2: The uniform extra fine mesh makes the blue numerical curve closer to the red analytical solution .

2-6-6 meshing is doable. It also gives inadequate results. (notice that it is consistent with the 3-6-6 meshing. Consistency is not the only indicator of a reliable mesh.)

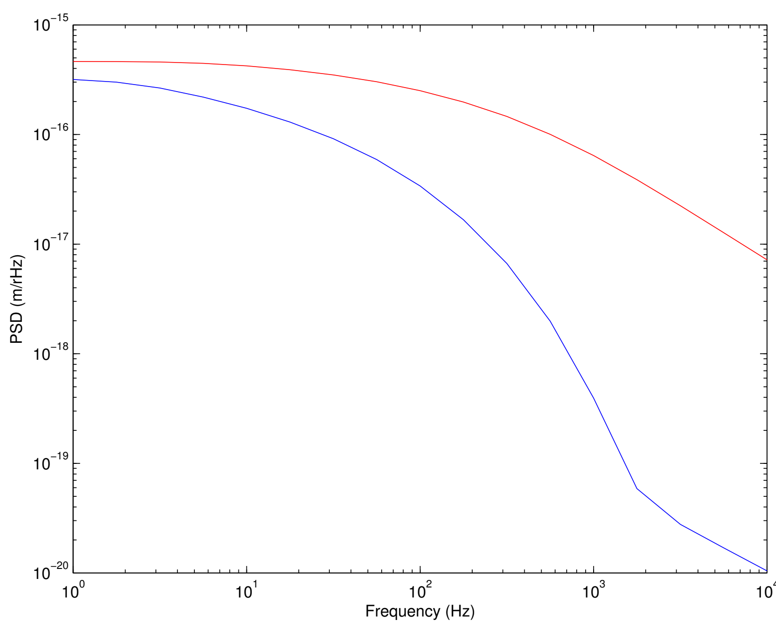

Figure 3: The coarse-extra fine sandwich[meshing for the blue numerical curve is also not adequate for $r_0 = 1$ mm.

Furthermore, a 2-3-3 mesh is also off.

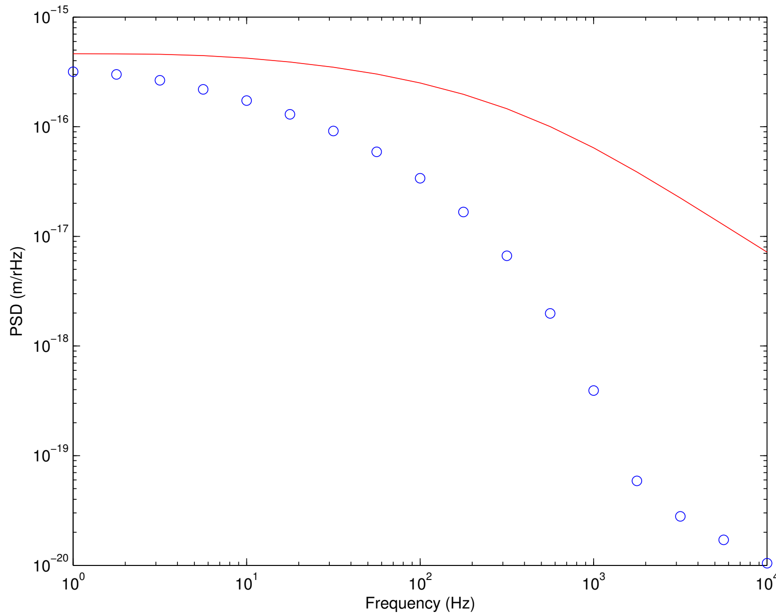

Figure 4: The finer-extra fine sandwich meshing for the blue numerical curve is also not adequate for $r_0 = 1$ mm. The sandwich curve appears to give the same result, yet all of them appear to be off.

Here's a uniform finer mesh comparison.

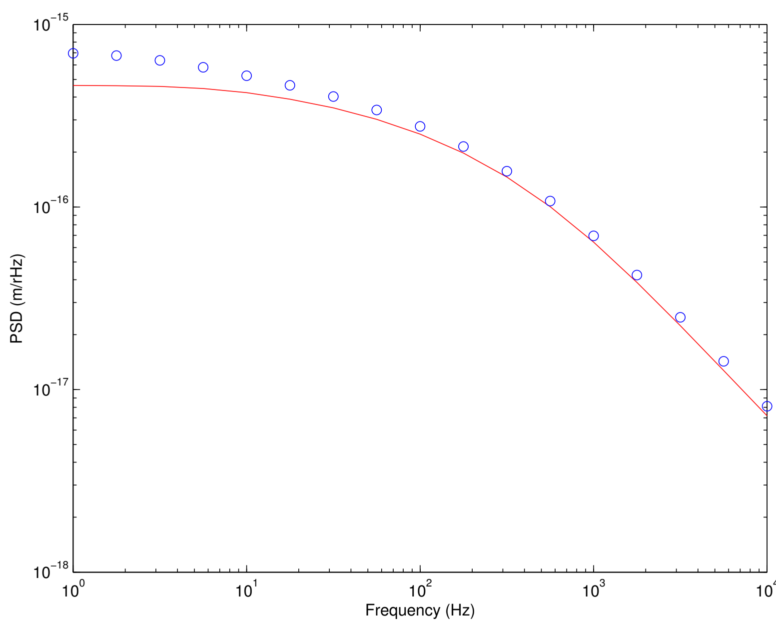

Figure 5:

I get this when I reduce the beam radius by half:

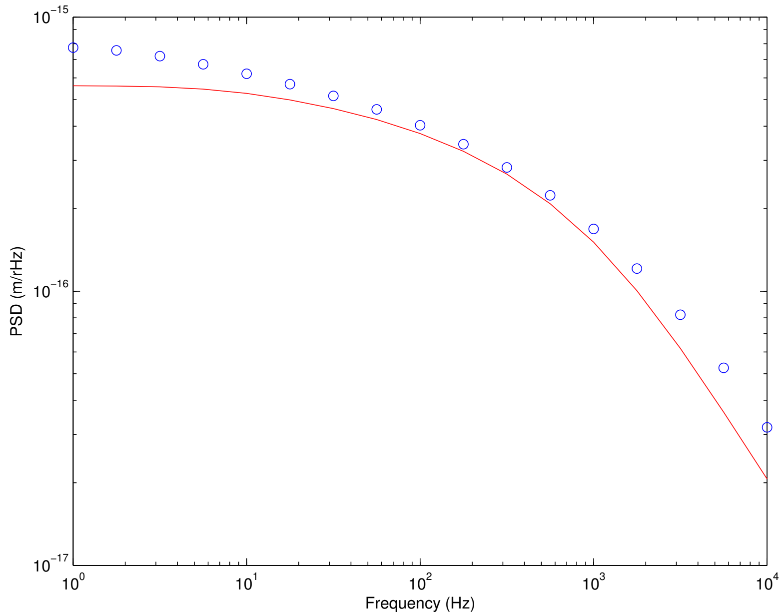

Figure 6: The finer mesh numerical solution is further off from the numerical solution when the beam radius is cut in half.

To get an idea of which Fourier components are important, I split up the analytical fourier coefficient as $d_{lmn} = d_{lm}d_n$. The d_n component comes purely from the z component, while the d_{lm} comes purely from the xy components. This gave the following plots. The z component is strongest initially, but it falls off to a constant, *nonzero* value. The xy components appear to have constant values, which is not good for truncating the Fourier Series.

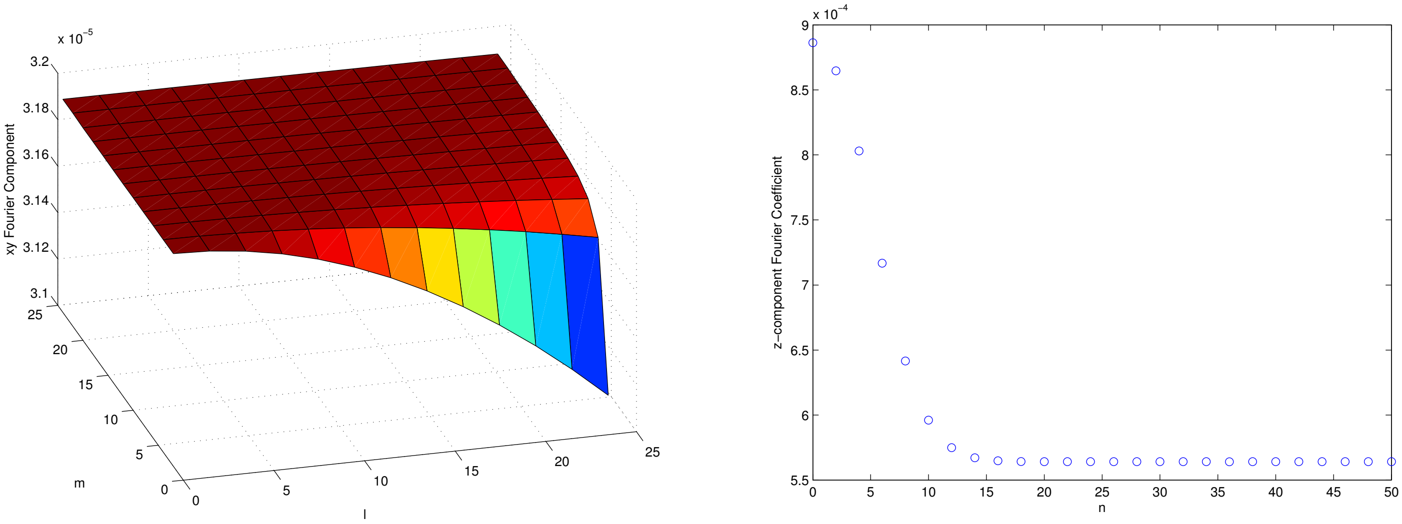

Figure 7: Fourier Components. The d_{lm} is plotted on the left, while the d_n is on the right. The fact that these coefficients don't fall quickly to zero is not a good sign for terminating the Fourier Series.