July 10, 2014

My plan has been to replicate Duan's numerical results presented in his paper (section V). I am trying to match Duan's analytical expression with Heinert's analytical expression. The calculation is of the thermoconductive (TE + TR) phase noise

$$S_\phi(f) = \frac{4\pi^2}{\lambda^2} H^2 (\beta + n\alpha)^2 S_{\delta T}(f),$$

which requires some rescaling of Heinert's TR displacement noise. (I also needed to divide Heinert's expression by 4π to match the Fourier Transform convention.) Duan's analytical expression for S_ϕ then amounts to

$$S_\phi(\omega) = \frac{2Hk_B T^2 (\beta + n\alpha)^2}{\lambda^2 r_0^4 \kappa} \int_1^\infty d\zeta' \int_1^\infty d\zeta \frac{e^{i\psi_0(\zeta - \zeta')}}{\zeta^2 \zeta'^2} \int_0^R r^3 \exp\left[-\frac{r^2}{r_0^2}\left(\frac{1}{\zeta} + \frac{1}{\zeta'}\right)\right],$$

where $\psi_0 = \omega r_0^2/(4D) = \pi f r_0^2/(2D)$ and $D = \kappa/C_p$. It turns out that an additional factor of 2 multiplies this S_ϕ expression because Duan's Fourier Transform only takes into account positive frequencies. There are also negative frequencies that occur in equal amplitude.

This integral was evaluated in Mathematica due to numerical noise in MATLAB's calculation. The calculation in Mathematica was very slow, so the upper limits on ζ and ζ' were truncated. The following plots show the resulting noise profile agreements for two different upper limits.

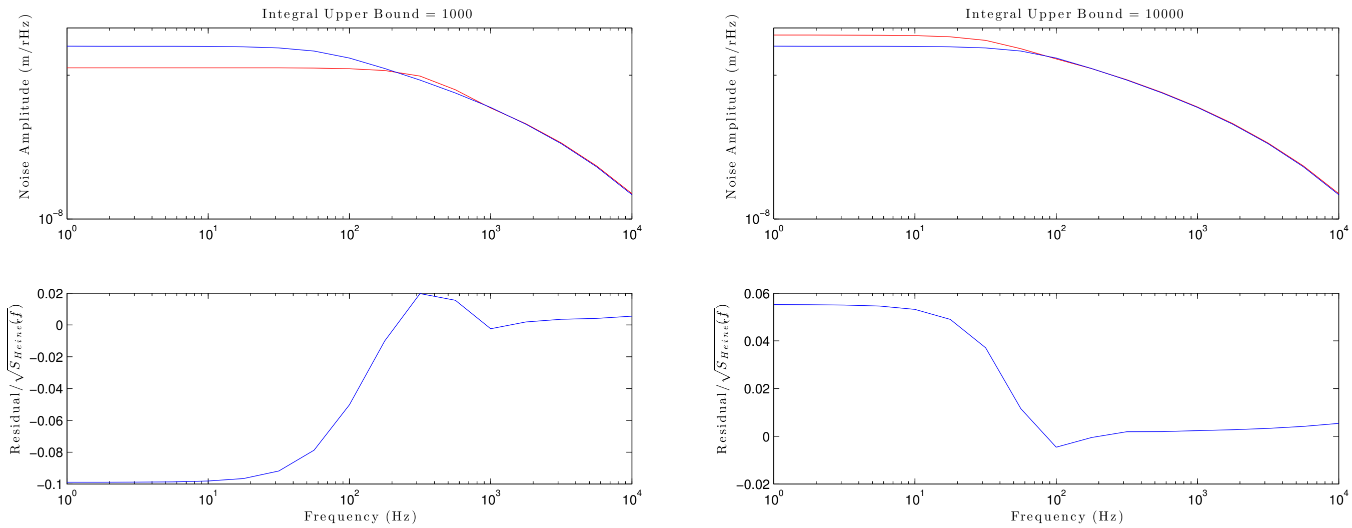

Figure 1: Phase Noise Amplitude comparison for Duan's numerical case, using Heinert's equation (blue) and Duan's equation (red). Duan curve gets closer to the Heinert curve when the upper bound is increased. From this increased agreement, I will assume that the plots indeed give the same result. This means that when COMSOL is used for Duan's case, Heinert's curve can be used as the reference (which runs faster).