June 27, 2014

My goal has been to verify Heinert's model in COMSOL, using a stationary harmonic analysis. It turns out that the Heat Transfer module is not capable of doing a harmonic analysis, so I am instead using the COMSOL "coefficient form module." The general form is given as

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f$$

Since I intend to solve

$$i\omega C_p T - \kappa \nabla^2 T = i\omega A e^{-r^2/r_0^2}$$

the only nonzero terms are $a = i\omega C_p$, $c = \kappa$, and $f = i\omega A e^{-r^2/r_0^2}$. I am using a 1D axisymmetric model, with a zero flux boundary condition at $r = R$. Being sure to split the solution for u into real and imaginary parts, the plot for $\text{Re}(u)$ gives nothing. Why is this?

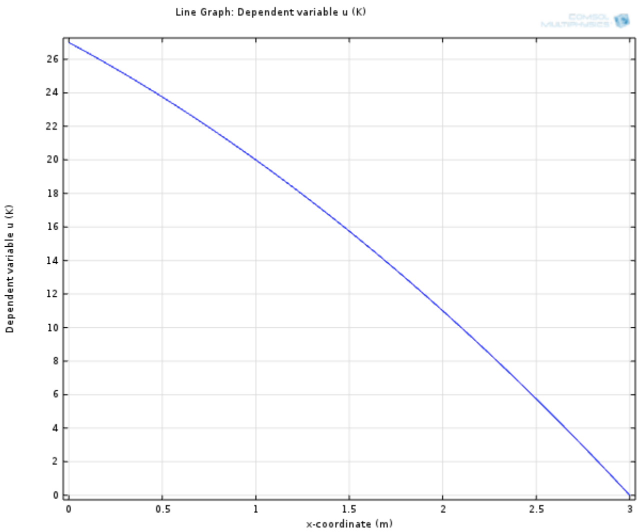
To try to get a better grip with COMSOL, I have been trying simpler cases, comparing an analytical result with COMSOL's result. For example consider a bar of length L, subject to a constant power input P_0. Then in the steady state, the heat equation is

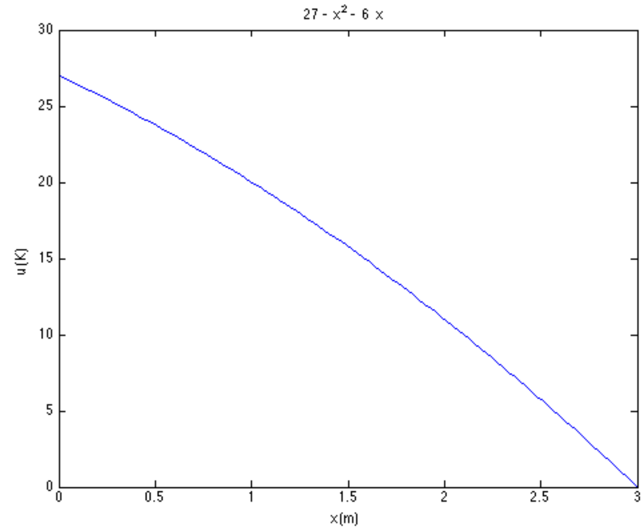
$$\kappa \frac{d^2 u}{dx^2} = -P_0 \Rightarrow u(x) = -\frac{P_0}{2\kappa} x^2 + ax + b$$

For clarity on the units, $[P_0] = W/m^3$ and $[\kappa] = W/m/K$.Here, we subject the bar to the boundary conditions $u(L) = 0$ and $u'(0) = -P_0 L/\kappa$ (this latter condition says that a heat flux of $-P_0 L$ is entering the rod (I'm not quite sure how to explain the negative sign). These conditions give

$$u(x) = -\frac{P_0}{2\kappa} x^2 - \frac{P_0 L}{\kappa} x + \frac{3 P_0 L^2}{2\kappa}$$

I have this plotted in MATLAB to compare with COMSOL's result. In COMSOL, I used a "Dirichlet Boundary Condition" at $x = L$ and a "Flux/Source" condition at $x = 0$





The analytic and COMSOL models show agreement for these cases. I would also like to try a harmonic excitation test case for a 1D bar, leading to something like

$$i\omega C_p T - \kappa \frac{d^2T}{dx^2} = i\omega q_0$$

subject to $T'(x = L) = -q_0\omega L/\kappa$ and $T(x = 0) = 0$. I can find the general solution here, but I'm having trouble applying the boundary conditions; I appear to be getting transcendental equations.