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My goal is now to verify some of the Heinert 2011 numerical results in COMSOL. In COMSOL, I use a 1D axially symmetric model because I am assuming Heinert's simple model. In this model, the heat injection does not depend on distance along the cylinder. By contrast, Heinert's Advanced model accounts for standing waves in the cavity, which destroys the translational symmetry. To verify Heinert's results, I'm not sure if it's adequate to use the simple model.

In COMSOL, I am hoping to find the time-independent stationary solution. To solve this analytically, I take the heat equation

$$C_p\dot{T} - \kappa\nabla^2 T = \dot{q}(\vec{r}, t),$$

where q is the injected heat source. Assuming the heat injection has the form $q(\vec{r}, t) = q(\vec{r})e^{i\omega t}$, the temperature distribution in the steady state should take on $T(\vec{r}, t) = T(\vec{r})e^{i\omega t}$, which yields

$$i\omega C_p T - \kappa\nabla^2 T = i\omega q(\vec{r})$$

In COMSOL, I am trying a heat injection of the form $iA\exp(-r^2/r_0^2)$, where r_0 is the beam radius. With this complex form, I just get a flat line for the temperature curve. Moreover, I am not sure that COMSOL is solving the right equation. COMSOL's time dependent heat equation has the form

$$C_p\dot{T} + \rho C_p\vec{u}\cdot\nabla T - \nabla\cdot(\kappa\nabla T) = \dot{q}(\vec{r}, t)$$

When I select in COMSOL 'stationary solution', the equation becomes

$$\rho C_p\vec{u}\cdot\nabla T - \nabla\cdot(\kappa\nabla T) = \dot{q}$$

In other words, the time-dependence is simply ignored instead of being converted to complex form (I don't know what the $\rho C_p\vec{u}\cdot\nabla T$ term represents. C_p is heat capacity at constant pressure, ρ is density, but I'm not sure about $\vec{u}$).

On another note, it looks like it is possible to define the material properties as a function of temperature, which may come in handy.