Sensitivity of Eigenfrequency Temperature Probe

Oscillator

We plan to track the eigenfrequency of an oscillator to measure its temperature. The oscillator is a 2'' silicon wafer. According to the literature [1], the first eigenmode of a homogeneous beam of silicon with uniform cross section is given by

$$\omega_1 = (1.875)^2 \frac{t}{L^2} \sqrt{\frac{E}{12\,\rho}}$$

t is the thickness of the beam, L the length, E the Young's modulus, ρ the density. For the case of a silicon disk, the dependence will be analogous, where the only component with a strong temperature dependence is E. Therefore, consider only

$$\omega \propto \sqrt{E}$$

This is why it is best to track the temperature using as high an eigenfrequency as possible; if we have some limiting frequency resolution, then we'd best have as large an absolute frequency shift as possible, and since E is a material property the only way we can do that is if the proportionality constant in the equation above to is large. To see this more explicitly:

Resolution

Let the eigenfrequency at the temperature of the zero crossing of silicon's coefficient of thermal expansion be ω_0, and let its Young's modulus be E_0. Then the eigenfrequency at some temperature T is

$$\omega(T) = \frac{\omega_0}{\sqrt{E_0}} \sqrt{E(T)}$$

Therefore,

$$\frac{d\omega}{dT} = \frac{\omega_0}{2\sqrt{E_0}} \frac{dE/dT}{\sqrt{E(T)}}$$

Material Properties

Now all we need to plug in are some material properties, conveniently located in the Voyager white paper documents 'MaterialProperties_and_Constants'

$$\frac{dE}{dT} = -\left(0.0158\, e^{\frac{-317}{T}} + \frac{0.0158 * 317}{T} e^{\frac{-317}{T}}\right) \frac{\text{GPa}}{K}$$

$$E(T) = \left(167.5 - 0.0158 * T * e^{\frac{-317}{T}}\right) \frac{\text{GPa}}{K}$$

Sensitivity Plot

In[56]:=
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e[T_] = 167.5 - 0.0158 * T * E^(-317/T)
dEdT[T_] = -0.0158 E^(-317/T) - ((0.0158 * 317)/T) E^(-317/T)
ω0 = 30 000;
E0 = e[123];
Plot[ω0/(2 Sqrt[E0]) * dEdT[T]/Sqrt[e[T]], {T, 77, 350},
 PlotLabel → "Sensitivity of 30kHz Eigenfrequency Temperature Probe",
 AxesLabel → {"T (K)", "Sensitivity (Hz/K)"}]
Plot[1/(2 Sqrt[E0]) * dEdT[T]/Sqrt[e[T]], {T, 77, 350},
 PlotLabel → "Fractional Eigenfrequency Temperature Probe Sensitivity",
 AxesLabel → {"T (K)", "Sensitivity (/K)"}]
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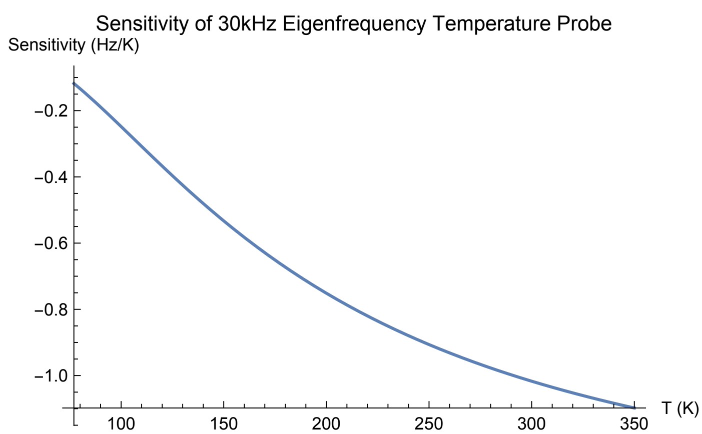
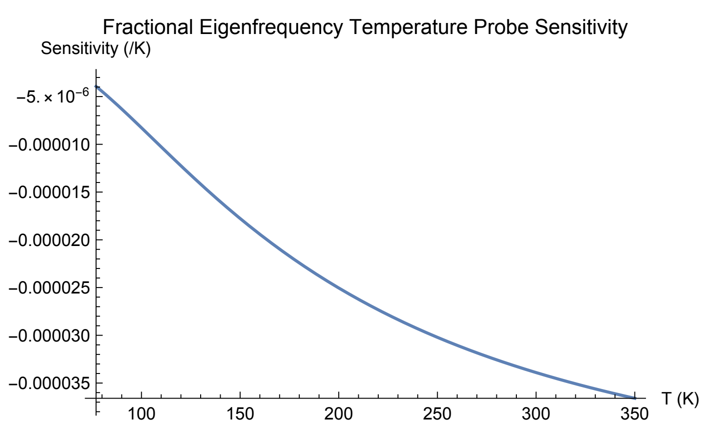
Out[56]= $167.5 - 0.0158\, e^{-317/T}\, T$

Out[57]= $-0.0158\, e^{-317/T} - \frac{5.0086\, e^{-317/T}}{T}$

Out[60]=



Out[61]=



As a sanity check, I know that when I modelled this in COMSOL I observed a ~10Hz frequency shift when I went from 300K to ~100K for a ~1kHz mode (don’t recall the exact numbers, could have been a factor away from this but it wasn't 1Hz shift and it wasn't 100Hz; anyway I think it's in an elog). That gives us something like a 1.5Hz/K expected sensitivity for 30kHz if we apply the proportional scaling, which is kind of close to what we get if a little optimistic. I think I quoted something like 1Hz/K sensitivity in the last group meeting, and it looks like that will only be true near room temperature, we'll have about half that sensitivity near 123K.

[1] Gysin, U. et al, Temperature dependence of the force sensitivity of silicon cantilevers

In[68]:=
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ω0 = 30000;
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$$\frac{\omega 0}{2\sqrt{E0}} * \frac{dEdT[123]}{\sqrt{e[123]}}$$

Out[69]= -0.384933