EOAM phenomenology

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We consider EOAMs of the New Focus 410x type. These consist of two crystals displaying the electro-optic effect, mounted at 45° relative to each other. Suppose the first has a length ℓ_1 and is mounted so that its ordinary axis is $\hat{\mathbf{o}}_1 = \hat{\mathbf{a}} = (\hat{\mathbf{x}} + \hat{\mathbf{y}})/2^{1/2}$, and its extraordinary axis is $\hat{\mathbf{e}}_1 = \hat{\mathbf{b}} = (-\hat{\mathbf{x}} + \hat{\mathbf{y}})/3^{1/2}$. Similarly, the second has a length ℓ_2 and is mounted so that $\hat{\mathbf{o}}_2 = -\hat{\mathbf{b}}$ and $\hat{\mathbf{e}}_2 = \hat{\mathbf{a}}$.

A field $\mathbf{E} = E_a\hat{\mathbf{a}} + E_b\hat{\mathbf{b}}$ is incident on the input of the EOAM. The accumulated phase shifts at the EOAM output are (neglecting free-space propagation)

$$\phi_a = \frac{2\pi}{\lambda_0}\left[n_\mathrm{o}(V_1)\ell_1 + n_\mathrm{e}(V_2)\ell_2\right] \tag{1a}$$

$$\phi_b = \frac{2\pi}{\lambda_0}\left[n_\mathrm{e}(V_1)\ell_1 + n_\mathrm{o}(V_2)\ell_2\right]. \tag{1b}$$

We assume $V_1 = -V_2 = V$. The behavior of the ordinary and extraordinary indices are $n_\mathrm{o,e}(V) = n^{(0)}_\mathrm{o,e} + n'_\mathrm{o,e}V$.[1] Then the phase difference $\phi = \phi_b - \phi_a$ is

$$\phi = \frac{2\pi}{\lambda_0}\left[\left(n^{(0)}_\mathrm{o} - n^{(0)}_\mathrm{e}\right)(\ell_2 - \ell_1) - \left(n'_\mathrm{o} - n'_\mathrm{e}\right)V\,(\ell_1 + \ell_2)\right] \tag{2}$$

$$= \phi_\mathrm{DC} + \phi'\,V. \tag{3}$$

We now suppose $\mathbf{E}_\mathrm{in} = E_1\hat{\mathbf{y}} = E_0(\hat{\mathbf{a}} + \hat{\mathbf{b}})/2^{1/2}$. Then the output is

$$\mathbf{E}_\mathrm{out} = \frac{E_0}{2^{1/2}}\left(\hat{\mathbf{a}} + \mathrm{e}^{\mathrm{i}\phi}\,\hat{\mathbf{b}}\right) \tag{4a}$$

$$= \frac{E_0}{2}\left[(1 - \mathrm{e}^{\mathrm{i}\phi})\hat{\mathbf{x}} + (1 + \mathrm{e}^{\mathrm{i}\phi})\hat{\mathbf{y}}\right]. \tag{4b}$$

If the beam is then sent through a QWP whose Jones matrix is

$$\mathbf{Q}_{\pi/4} = \frac{1}{2^{1/2}}\begin{pmatrix}1 & \mathrm{i}\\ \mathrm{i} & 1\end{pmatrix}, \tag{5}$$

then the x component of the output field is

$$E_x = \frac{E_0}{2^{3/2}}\left[(1 + \mathrm{i}) - (1 - \mathrm{i})\mathrm{e}^{\mathrm{i}\phi}\right], \tag{6}$$

and hence the output power is

$$P_x = |E_x|^2 = \frac{E_0^2}{8}\left[4 + 2\mathrm{i}(\mathrm{e}^{\mathrm{i}\phi} - \mathrm{e}^{-\mathrm{i}\phi})\right] \tag{7}$$

$$= \frac{P_0}{2}\left[1 - \sin\left(\phi_\mathrm{DC} + \phi'\,V\right)\right]. \tag{8}$$

[1] C. C. Davis, *Lasers and Electro-optics* 2nd ed., eq. 18.44.

If instead we use a QWP at some other angle, we have

$$\mathbf{Q}_\theta = \frac{1}{2^{1/2}} \begin{pmatrix} 1 - \mathrm{i}\cos 2\theta & \sin 2\theta \\ \sin 2\theta & 1 + \mathrm{i}\cos 2\theta \end{pmatrix}, \tag{9}$$

and then

$$P_x = \frac{P_0}{2} \left[1 - \cos^2(2\theta) \cos\phi - \sin(2\theta) \sin\phi \right]. \tag{10}$$