PDH sensing of a birefringent Fabry–Pérot cavity

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We assume a simple phenomenological model of the reflectivity matrix for a birefringent cavity. We write

$$\mathbf{r}(\omega) = \begin{pmatrix} r_p(\omega) & 0 \\ 0 & r_s(\omega) \end{pmatrix}, \tag{1}$$

where r_p and r_s are the Fabry–Pérot reflectance functions for p- and s-polarized fields, respectively.[1]

We now suppose we implement PDH sensing using a PBS and a $\lambda/4$ plate. Then the light incident on the waveplate is $\mathbf{E}_0 = \begin{pmatrix} 1 & 0 \end{pmatrix}^T$. The Jones matrix describing the waveplate is

$$\mathbf{Q} = \frac{1}{2^{1/2}} \begin{pmatrix} 1+\mathrm{i} & 1-\mathrm{i} \\ 1-\mathrm{i} & 1+\mathrm{i} \end{pmatrix}. \tag{2}$$

It reflects from the cavity as $\mathbf{E}_1 = \mathbf{r}(\omega)\mathbf{E}_0$, and passes through the waveplate. The field heading back toward the PBS is therefore

$$\begin{aligned} \mathbf{E}_2(\omega) &= \mathbf{Q}\,\mathbf{r}(\omega)\,\mathbf{Q} \begin{pmatrix} E_0 \\ 0 \end{pmatrix} && (3) \\ &= \frac{1}{2} \begin{pmatrix} \mathrm{i}(r_p - r_s) & r_p + r_s \\ r_p + r_s & -\mathrm{i}(r_p - r_s) \end{pmatrix} \begin{pmatrix} E_0 \\ 0 \end{pmatrix} && (4) \\ &= \frac{E_0}{2} \begin{pmatrix} -\mathrm{i}(r_p - r_s) \\ r_p + r_s \end{pmatrix}. && (5) \end{aligned}$$

If the RFPD is placed on the reflection (s) port of the PBS, the sensed field is then

$$E_{\mathrm{PD}}(\omega) = \frac{1}{2} E_0(\omega) \left[r_p(\omega) + r_s(\omega) \right], \tag{6}$$

and hence the PDH demodulation will perform as usual: defining $r = (r_p + r_s)/2$, the PDH slope $\mathcal{D}$ is proportional to

$$\mathrm{Im}\left(r_0^* r_{+\Omega} - r_0 r_{-\Omega}^* \right), \tag{7}$$

where $r_0 = r(\omega_0)$ and $r_{\pm\Omega} = r(\omega_0 \pm \Omega)$.

[1] p and s are taken in the usual sense for tabletop optics.