Thermal Expansion Noise due to RIN

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1 Effect of RIN on frequency stabilization system

The laser power fluctuation absorbed on the coating will cause two types of noise. First is the thermal expansion which changes the position of the mirror surface. Second is thermal refractive which changes the phase of the reflected beam. Two noises are expected to cancel, not entirely at arbitrary frequency, each other. Thus, in this report, I calculate the noise due to thermal expansion to see the upper limit of noise from RIN. I will treat the multiple layer coatings as a single layer of the same thickness(4.4 um) which attached to the mirror substrate.

2 calculation

Since the spot size,w, on the mirror is small compared to the mirror size, we will assume that the mirror is half infinite. The spot size is 300 microns. The mirror is 1 inch in diameter and 1/4 inch thick.

The heat equation is

$$T_t = \frac{\kappa}{C} T_{xx} \tag{1}$$

We Laplace transform the heat equation.

$$L[T_t] = L[\alpha T_{xx}] \tag{2}$$

$$s\tilde{T} = \alpha \tilde{T}_{xx} \tag{3}$$

The solution to this differential equation is in the form of

$$\tilde{T}_1 = C_1 Exp[-x\sqrt{\frac{s}{\alpha_1}}] + C_2 Exp[+x\sqrt{\frac{s}{\alpha_1}}], x = [0, d] \tag{4}$$

in the first medium, and

$$\tilde{T}_2 = C_3 Exp[-x\sqrt{\frac{s}{\alpha_2}}], x = [0, \inf] \tag{5}$$

in the second medium.

Using the continuity of temperature at x = d in the first medium, and at x = 0 in the second medium, (those are the position at the connected surfaces,) we get

$$\tilde{T}_1(x = d) = \tilde{T}_2(x = 0), \tag{6}$$

and heat flow from the first medium equals to the heat flow into the second medium,

$$\kappa_1 \tilde{T}_{1,x}(x = d) = \kappa_2 \tilde{T}_{2,x}(x = 0). \tag{7}$$

The third boundary condition is

$$\kappa_1 \tilde{T}_{1,x}(x = 0) = L[-\delta I sin(wt)] = [Exc]. \tag{8}$$

This is the heat flux entering the mirror due to RIN. δI is the intensity absorbed on the coating surface. We denote it as an excitation for now.

Solving (8),(9), and (10), we get

$$C_1 = +\frac{1}{1 + e^{-2\theta} Q^{-1}} \sqrt{\frac{\alpha_1}{s}} \frac{[Exc]}{\kappa_1} \tag{9}$$

$$C_2 = -\frac{1}{1 + e^{2\theta} Q} \sqrt{\frac{\alpha_1}{s}} \frac{[Exc]}{\kappa_1} \tag{10}$$

$$C_3 = C_1 e^{-\theta} + C_2 e^{\theta} = (\frac{e^{-\theta}}{1 + e^{-2\theta} Q^{-1}} - \frac{e^{\theta}}{1 + e^{2\theta} Q}) \sqrt{\frac{\alpha_1}{s}} \frac{[Exc]}{\kappa_1}. \tag{11}$$

where

$$\begin{aligned} \alpha_i &= \frac{\kappa_i}{C_i} \\ \theta &= d\sqrt{\frac{\alpha 1}{s}}, \\ Q &= \frac{\lambda + 1}{\lambda - 1}, \\ \lambda &= \sqrt{\frac{\kappa_2 C_2}{\kappa_1 C_1}} * \end{aligned}$$

*I am not very careful about naming here. C1 and C2 are heat capacity of the coating and the substrate, respectively, not the constant in the equations above.

The results allow us to find the transfer function of our system.

$$\delta\tilde{T}(x,s) = TF(w) \times [Exc] \tag{12}$$

For now we are interested in the response at 10Hz.

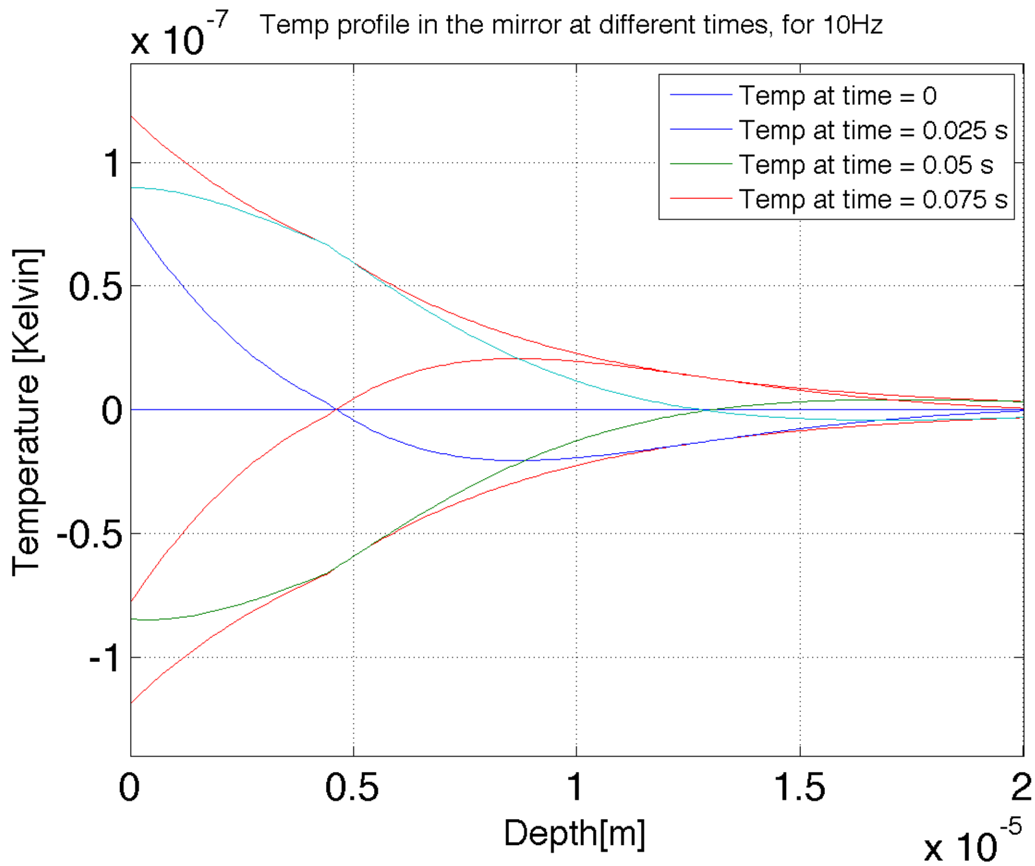

Figure 1: Temperature profile in the mirror from 10Hz heat wave. Y axis is temperature variation, X axis is depth inside the mirror

The actual temperature variation inside the mirror can be computed by multiplying the TF by the excitation amplitude δI, which is

$$\begin{aligned}
\delta I &= \frac{Pin}{\pi r_G^2}\frac{Finesse}{\pi}abs \times RIN \\
&= \frac{10[mW]}{\pi(300[um])^2}\frac{10^4}{\pi}5ppm \times 10^{-4} \\
&= 0.0563\frac{W}{m^2}.
\end{aligned}$$

After obtaining the temperature inside the mirror, we can calculate the thermal expansion of the mirror via a simple relation,

$$\Delta z = \alpha_{th}\Delta T L \quad (13)$$

$$= \Sigma\alpha_i\Delta T_i\Delta L_i \quad (14)$$

The thermal expansion coefficient, α_{th}, of bulk SiO2 is $0.51 \times 10^{-6}[\frac{1}{K}]$. To calculate the thermal expansion coefficient of the coating compound, we use the result from [102003], and get $\alpha_{th,coating}$ to be $5.8 \times 10^{-6}[\frac{1}{K}]$. The thermal expansion effect can be plotted as a function of time, see fig 3.

For a rough approximation $dz_{pk-pk} = 4 \times 10^{-18}$, and x2 for two mirrors, then

$$\begin{aligned}
df &= 2\frac{dz}{L} \times \frac{c}{\lambda} \\
&= 0.012Hz.
\end{aligned}$$

The result is very small from what we expect.

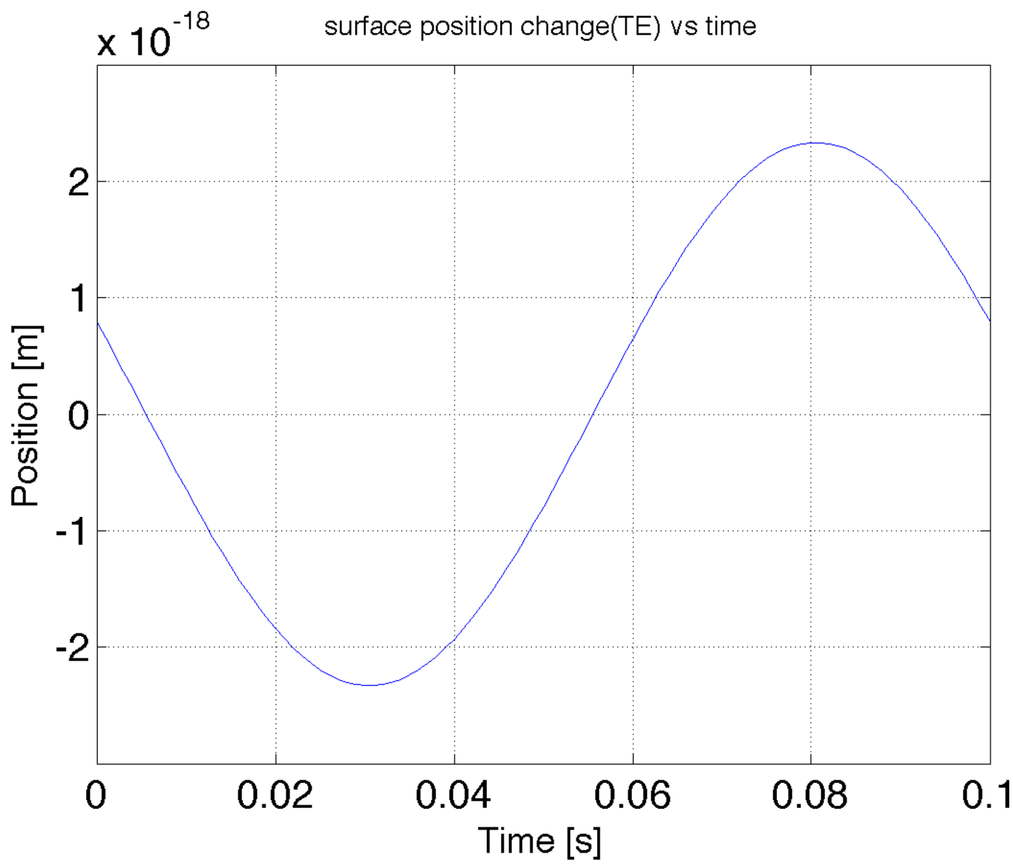

Figure 2: effect from thermal expansion, displacement as a function of time