Prep

```
In[301]:= g = 9.8 m / s^2;
mm = 0.001 m;
ρ = 2200 kg / m^3;
```

3 inch glass optics

```
In[304]:= R = d / 2;
d = 3 * 25.4 mm;
h = 25.4 mm;
M = ρ π R^2 h;
```

In[308]:= $Iz = \frac{M R^2}{2}$

$Ix = M\left(\frac{R^2}{4} + \frac{h^2}{12}\right)$

Out[308]= $0.000184959\ \mathrm{kg\ m^2}$

Out[309]= $0.00010618\ \mathrm{kg\ m^2}$

```
In[310]:= dc = 0.6578 * 25.4 mm;

In[311]:= d1 = 0; (* wire gap. 0 for the two wire case *)
y0 = 25.4 mm * 1.5; (* optic radius *)
dc = 0.6578 * 25.4 mm; (* wire distance at the upper clamp *)
y1 = y0 - dc / 2;
z0 = 0.9 mm; (* Clamping point height relative to the height of the COM *)
R0 = 9.7455 * 25.4 mm; (* Pendulum vertical length *)
```

Spring Matrix Elements

In[317]:= `Kxx = M g / R0;`

In[318]:= $K\xi\xi = \frac{4 k\, d1^2\, y1^2}{l^2} + \frac{M g\, d1^2\, R0}{l^2} + \frac{M g\, y0\,(y0 - y1)}{R0};$

In[319]:= $K\eta\eta = \frac{4 k\, d1^2\, R0^2}{l^2} + \frac{M g\,(z0\,(R0 + z0)\, l^2 + d1^2\, y1^2)}{l^2\, R0};$

Resonant Freqs

In[320]:= `ω / (2 π) /. Solve[Kxx - ω^2 M == 0, ω] /. s → 1 / Hz`

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[320]= {-1.00142 Hz, 1.00142 Hz}

```
In[321]:= (* Yaw *)
ω / (2 π) /. Solve[Kξξ - ω^2 Ix == 0, ω] /. s → 1 / Hz
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[321]= {-0.875249 Hz, 0.875249 Hz}

In[322]:= `ω / (2 π) /. Solve[Kηη - ω^2 Ix == 0, ω] /. s → 1 / Hz`

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[322]= {-0.733582 Hz, 0.733582 Hz}

2”->3” Metal Sleeve

In[323]:=
```
(* From Elog 16137 *)
Lyy = 96.567428 kg * mm^2;
Lzz = 94.353847 kg * mm^2;
Lxx = 175.108843 kg * mm^2;
```

In[326]:=
```
R = d / 2;
d = 3 * 25.4 mm;
h = 25.4 mm;
M = 197.4 / 1000 kg;
(*ρ=2200kg/m^3;*)
```

In[330]:=
```
dc = 0.6578 * 25.4 mm;
```

In[331]:=
```
d1 = 0; (* wire gap. 0 for the two wire case *)
y0 = 25.4 mm * 1.5; (* optic radius *)
dc = 0.6578 * 25.4 mm; (* wire distance at the upper clamp *)
y1 = y0 - dc / 2;
z0 = Z0 mm; (* Clamping point height relative to the height of the COM *)
R0 = 9.7455 * 25.4 mm; (* Pendulum vertical length *)
```

Spring Matrix Elements

In[337]:=
```
Kxx = M g / R0;
```

In[338]:=
$$K\xi\xi = \frac{4\,k\,d1^2\,y1^2}{l^2} + \frac{M\,g\,d1^2\,R0}{l^2} + \frac{M\,g\,y0\,(y0 - y1)}{R0};$$

In[339]:=
$$K\eta\eta = \frac{4\,k\,d1^2\,R0^2}{l^2} + \frac{M\,g\,(z0\,(R0 + z0)\,l\verb|^|2 + d1\verb|^|2\,y1\verb|^|2)}{l\verb|^|2\,R0};$$

Resonant Freqs

In[340]:=
```
ω / (2 π) /. Solve[Kxx - ω^2 M == 0, ω] /. s → 1 / Hz
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[340]= {-1.00142 Hz, 1.00142 Hz}

In[341]:=
```
(* Yaw *)
ω / (2 π) /. Solve[Kξξ - ω^2 Lzz == 0, ω] /. s → 1 / Hz
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[341]= {-0.817183 Hz, 0.817183 Hz}

In[342]:=
```
sol = (ω / (2 π) /. Solve[Kηη - ω^2 Lyy == 0, ω] /. s → 1 / Hz) [[2]]
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[342]= $0.000452765\,\text{Hz}\,\sqrt{2.47536\times10^{6}\,Z0 + 10\,000.\,Z0^2}$

In[343]:=
```
Solve[sol == 0.734 Hz, Z0][[2]]
```

Out[343]= {Z0 → 1.0572}

Center of Mass: 1.1mm lower than the height of the clamping point