## 1 Mirror and Cavity Equations

Each mirror $i$ has reflectivity $R_i$, transmissivity $T_i$, and dissipative losses $\mathcal{L}_i$, such that

$$R_i = 1 - T_i - \mathcal{L}_i \tag{1}$$

This yields a reflection coefficient $r_i$ of

$$r_i = \sqrt{R_i} \approx 1 - \frac{T_i}{2} - \frac{\mathcal{L}_i}{2} \tag{2}$$

and a frequency ($\nu$) dependent compound cavity reflectivity

$$r_c(\nu) = \frac{r_1 - r_2({r_1}^2 + {t_1}^2)\exp(-i2\pi\frac{\nu}{\mathrm{FSR}})}{1 - r_1 r_2 \exp(-i2\pi\frac{\nu}{\mathrm{FSR}})} \tag{3}$$

## 2 Reflected Power $P_m$ with ETM Misaligned

When ETM is misaligned, the incicent beam sees only ITM, thus the reflected power is

$$P_m = (1 - T_1 - \mathcal{L}_1)P_0 \tag{4}$$

## 3 Reflected Power $P_l$ with ETM Aligned, Cavity Locked

On resonance we can set $\nu = 0$ and find

$$r_l = \frac{r_1 - r_2({r_1}^2 + {t_1}^2)}{1 - r_1 r_2} \approx \frac{1 - \frac{T_1}{2} - \frac{\mathcal{L}_1}{2} - \left(1 - \frac{T_2}{2} - \frac{\mathcal{L}_2}{2}\right)(1 - \mathcal{L}_1)}{\mathcal{L}_t/2} \tag{5}$$

which further simplifies to the well known expression

$$r_l = \frac{-\frac{T_1}{2} + \frac{\mathcal{L}_1}{2} + \frac{T_2}{2} + \frac{\mathcal{L}_2}{2}}{\mathcal{L}_t/2} = \frac{\frac{\mathcal{L}_t}{2} - T_1}{\mathcal{L}_t/2} = 1 - 2\frac{T_1}{\mathcal{L}_t} \tag{6}$$

The total loss $\mathcal{L}_t = T_1 + T_2 + \mathcal{L}_{\mathrm{diss}}$ is a combination of transmissive and total dissipative loss $\mathcal{L}_{\mathrm{diss}} = \mathcal{L}_1 + \mathcal{L}_2$ in the cavity. The reflected DC power on resonance is

$$P_l = \left(1 - 2\frac{T_1}{\mathcal{L}_t}\right)^2 P_0 = \left(1 - 4\frac{T_1}{\mathcal{L}_t} + 4\frac{{T_1}^2}{{\mathcal{L}_t}^2}\right) P_0 = \left(1 - 4\frac{T_1}{{\mathcal{L}_t}^2}\underbrace{(\mathcal{L}_t - T_1)}_{\mathcal{L}_{\mathrm{diss}} + T_2}\right) P_0 \tag{7}$$

Using $T_1 \gg T_2, \mathcal{L}_1, \mathcal{L}_2$ we can approximate $\frac{T_1}{{\mathcal{L}_t}^2} \approx \frac{1}{T_1}$ and find

$$P_l = \left(1 - 4\frac{\mathcal{L}_{\mathrm{diss}} + T_2}{T_1}\right) P_0 \tag{8}$$

## 4 Adjusting for Mode-Matching Defect $\alpha$

In the misaligned ETM case there is no cavity, and the reflection is off ITM alone, such that

$$P_m = (1 - T_1 - \mathcal{L}_1)P_0 \tag{9}$$

holds. In the aligned, resonant case we have to account for reduced visibility $\beta = 1 - \alpha$ due to the the parasitic higher-order-mode content $\alpha$ which does not contribute the to incient power used in the previous section. We obtain

$$P_l = \alpha P_0 + (1 - \alpha)\left(1 - 4\frac{T_1}{\mathcal{L}_t} + 4\frac{{T_1}^2}{{\mathcal{L}_t}^2}\right) P_0 = \left(1 - 4\beta\frac{\mathcal{L}_{\text{diss}} + T_2}{T_1}\right) P_0 \tag{10}$$

## 5 Adjusting for Power Loss to Sidebands

The phase modulation moves light power from the carrier to sidebands which will not resonate, and thus have the same effect as $\beta$. The series expansion of a single phase modulation term reads

$$E_i = E_0 \mathrm{e}^{i\omega t} \mathrm{e}^{im\Omega t} = E_0 \mathrm{e}^{i\omega t} \sum_{n=-\infty}^{\infty} J_n(m) \mathrm{e}^{i\Omega t} = E_0 \mathrm{e}^{i\omega t} J_0(m) + \text{sideband terms} \tag{11}$$

We can account for the corresponding power loss in the to-be-resonant carrier due to the dual modulation by replacing $\beta$ with an *effective visibility* $\gamma$

$$\beta \rightarrow \gamma = \beta \times [J_0(m_1)]^2 \times [J_0(m_2)]^2 \tag{12}$$

## 6 Method

1. Need to know $\alpha$, $m_1$, $m_2$
2. Measure $P_m$ and $P_l$
3. Calculate cavity losses from $\mathcal{L}_{\text{diss}} = \frac{T_1}{4\gamma}\left[1 - \frac{P_l}{P_m} + T_1\right] - T_2$