Measure of the Macroscopic Cavity Lengths of the Caltech 40m Interferometer

Alberto Stochino, Koji Arai

June 23, 2008

Abstract

We measured the lengths of both the X and Y arm cavities of the LIGO 40m Interferometer at Caltech by means of the RF modulation with a precision of 0.2.

1 The technique

Suppose we have a laser carrier of wavelength λ_c resonating in a Fabry-Perot cavity of length L. Suppose now we slowly change the laser frequency and at the same time we move the end mirror to keep the cavity on resonance so that the number of half wavelengths contained in the entire length does not change. If λ_s is the final wavelength of the laser, the mirror has to be displaced by ΔL:

$$\Delta L = \left[\left(\frac{L}{\lambda_c/2} \right) \cdot \frac{\lambda_s}{2} - L \right].$$

If, instead of variating the frequency, we suppose to have two laser components of wavelength λ_c and λ_s coexisting in the cavity, then ΔL is the displacement of the mirror necessary to makeλ_s resonant with the same number of half wavelengths. The case is analogous to that in which λ_c and λ_s are the carrier and the sideband wavelength of a modulated laser.

Starting when the carrier is resonant, we want to extend the cavity length displacing the end mirror by ΔL until the sideband becomes resonant. If, while sweeping the mirror position, we look at the transmitted power through the mirror or at the demodulated reflected power, we observe peaks when either one of the component is resonant. Naming ΔL_{fsr} the distance between two adjacent peaks of the carrier, this is equal to half wavelength of the carrier and we have:

$$\frac{\Delta L}{\Delta L_{fsr}} = \frac{\left(\frac{L}{\lambda_c/2} \right) \cdot \frac{\lambda_s}{2} - L.}{\lambda_c/2} = \frac{\nu_m}{\nu_{fsr}} \cdot \frac{\nu_c}{\nu_s} \approx \frac{\nu_m}{\nu_{fsr}}$$

where ν_m is the modulation frequency and the last approximation holds for the case of a RF modulation. From the measurement of $\Delta L/\Delta L_{fsr}$, knowing the carrier and the sideband wavelength, we can calculate the free spectral range ν_{fsr} and then obtain the cavity length L as:

$$L = \frac{c}{2\nu_m} \frac{\Delta L}{\Delta L_{fsr}}.$$

Then the error on L is:

$$\delta L = \frac{c}{2\nu_m} \cdot \left(\frac{\delta\left(\Delta L\right)}{\Delta L_{fsr}} + \frac{\Delta L \cdot \delta\left(\Delta L_{fsr}\right)}{\Delta L_{fsr}^2} \right)$$

where δindicates the error on the single quantities.

2 The measurement

We cannot have direct measures of ΔL or ΔL_{fsr} with the needed resolution. However we can evaluate their ratio from that of the time intervals between the occurrences of the correspondent resonances on a time series plot of either the transmitted or the reflected power. The ratio of displacements and the ratio of time intervals turn out to be the same if the mirror moves at a constant velocity. If the movement of the mirror is a pendulum-like oscillation, we can assume the velocity as constant in a certain interval around the equilibrium point, where the acceleration is zero.

Two sidebands are introduced by the modulation of the laser. Figure 1 shows the series of a carrier peak followed by two sideband peaks. Since we approximately know that $L \approx 38.5m$ we expect to count 8 carrier modes in between the carrier and the sideband corresponding to the same longitudinal mode of the cavity. Looking at the time series plot, we always have two sideband resonances between two carriers. One is the lower sideband of a carrier mode located 8 carrier resonance peaks before in the time series, and the other is the upper sideband of a carrier mode located 8 carrier resonances after in the plot. Before measuring ΔL we have to distinguish the upper sideband from the lower one. The only way to decide is a posteriori, that is estimating the cavity length from both the resonances and choosing the one that gives the most reasonable value for the cavity length considering the value known for it.

We chose to read the positions of the peaks in the plot from the point in which the demodulated output crosses zero. A different measurement of ΔL_{fsr} was obtained for each sideband resonance averaging over the 8 intervals between the carrier resonances. That enabled us to better keep into account for the change of velocity of the mirror during time.

From the average on 12 measures, and using the values $\lambda c = 1064nm$, $\nu_m = 33.195439Mhz$ and $\delta\left(\Delta L\right) = \delta\left(\Delta L_{fsr}\right) = 5 \times 10^{-5}s$ for the parameters, we obtained the following numbers for the cavity lengths $L_x = (38.30/38.45 \pm 0.08)m$, $L_y = (38.16/38.70 \pm 0.08)m$ in which the the two possibilities are obtained depending on which of the sideband resonance belongs to the lower sideband.

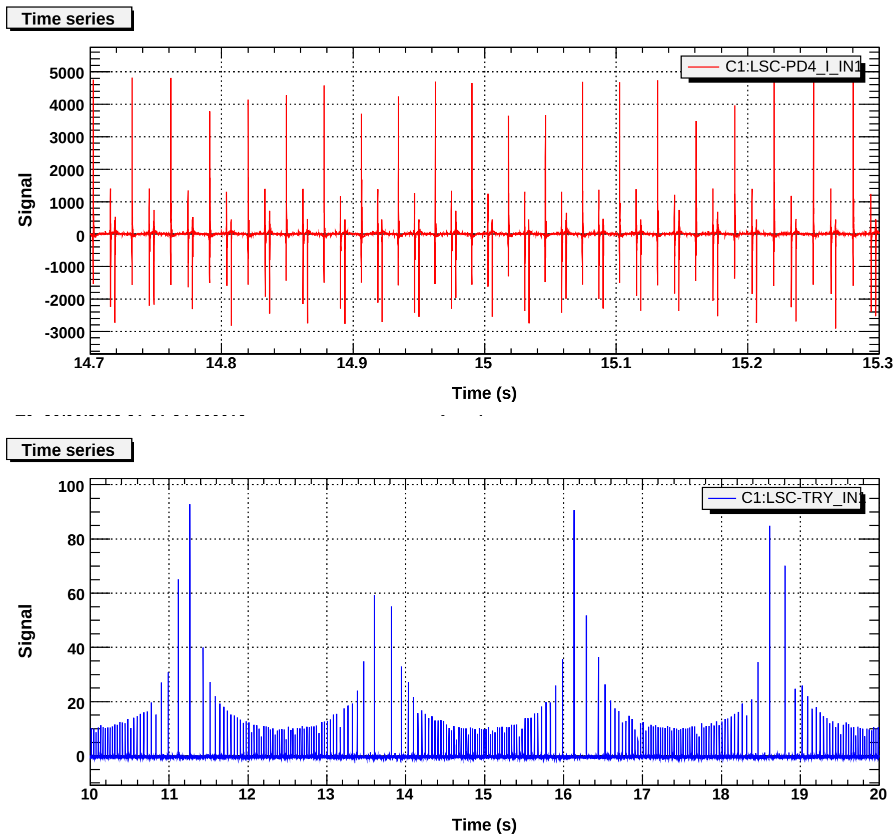

Figure 1: Time series plot of the demodulated reflected output (upper figure) and of the transmitted output (lower figure). The change in amplitude well shown in the second plot is due to the different velocity of the mirror during time compared to a build-up time of the cavity of about 1 ms. We measure more intensity for the resonances in which the mirror stays for longer.

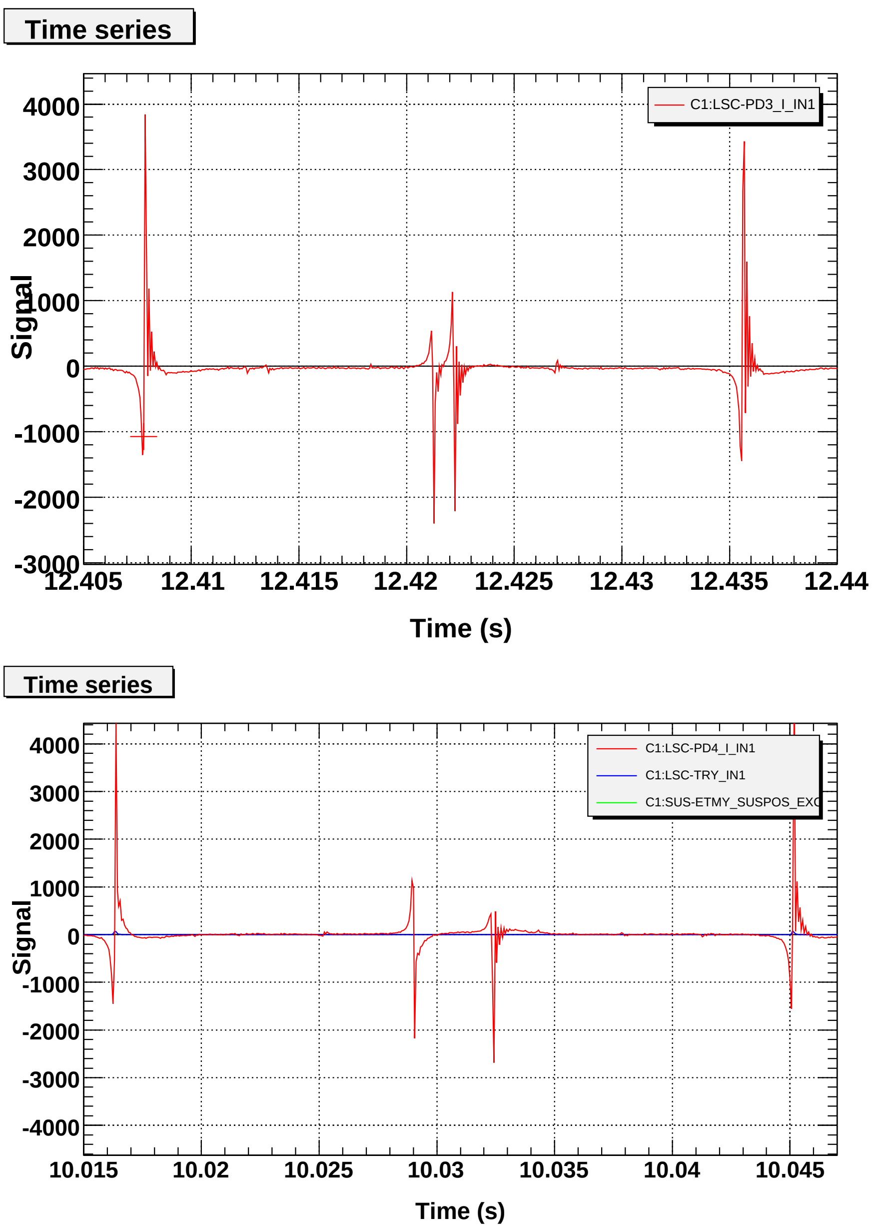

Figure 2: The plots in the figure show the zoom on a time interval of the demodulated output power of the X (upper plot) and the Y (lower plot) arm cavities. It is no possible a priori to distinguish which of the sideband resonances is the upper or the lower one. The separation between the sideband resonances is due to the different lenth of the to arm cavities,