Limits of the Noise Cancellation Ratio

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1 Introduction

Here, I will show how the noise cancellation ratio is limited by instrumental noise injected into the witness sensor or to the signal from which you want to substract the noise (such as seismic noise).

Here I assumed the algorithm is perfect and Wiener filter is ideal. Also I did not consider about feedback loop. Still it is reasonable for offline noise cancelling.

Wiener filter H minimizes the ensemble of residual error $e = d - Wx$. Here, d is the target signal before filtering (PMC control signal in this case), and x is the istrumental signal to be cancelled from the target signal (microphone signal in this case). When there is no noise, W is written as

$$W = \frac{P_{xs}}{P_{xx}}, \tag{1}$$

where P_{ik} is cross correlation of signals i and k.

2 Uncorrelated noise in primary input

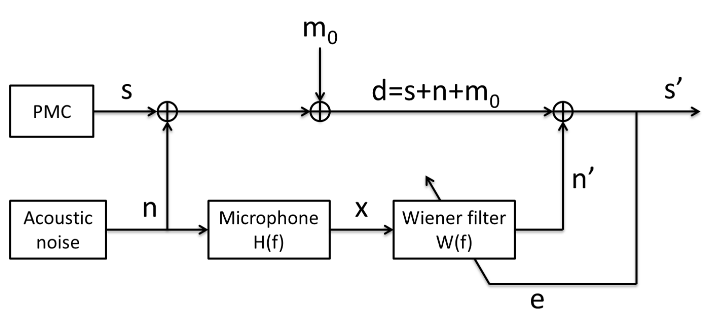

Figure 1: The block diagram of wiener filtering when the noise is injected just before the signal cancellation.

Let us consider what the wiener filter will be when noise m_0 is injected just just before noise cancellation point. m_0 considered to be mainly ADC noise. Note that this is not the noise in PMC such as seismic noise. Such noises are included in s.

In this case, the wiener filter is written as

$$W(z) = \frac{P_{xd}}{P_{xx}} \quad (2)$$

$$= \frac{P_{xs} + P_{xn} + P_{xm_0}}{P_{xx}} \quad (3)$$

$$= \frac{P_{xn}}{P_{xx}} \quad (4)$$

$$= \frac{1}{H(z)}. \quad (5)$$

Here we assumed that n, s, and m_0 is not correlated. The weiner filter does not change according to the noise in primary input.

3 Uncorrelated noise in reference input

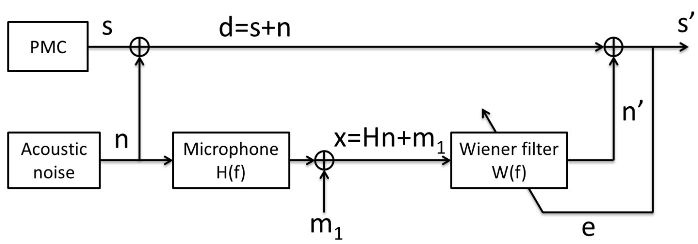

Figure 2: The block diagram of wiener filtering when the noise is injected into the enviromental noise monitor.

Second, I will describe about the wiener filter when the noise m_1 is injected into the environmental noise monitor, such as a microphone. m_1 includes the noise of microphone, pre-amp, and ADC noise.

The wiener filter will be

$$W(z) = \frac{P_{xd}}{P_{xx}} \quad (6)$$

$$= \frac{P_{(Hn)n}}{P_{(Hn+m_1)(Hn+m_1)}} \quad (7)$$

$$= \frac{P_{nn}H^*(z)}{P_{nn}|H(z)|^2 + P_{m_1m_1}}. \quad (8)$$

In this case, the wiener filter has been changed.

4 Noise cancellation rate

Here we consider the case where the both noise m_0 and m_1 are included. Since the original target signal is $d = s + n + m_0$, its PSD is

$$S_d = S_s + S_n + S_{m_0}. \quad (9)$$

The residual error is $e = d - Wx = (s + n + m_0) - W(Hn + m_1) = s + m_0 - Wm_1 + (1 - HW)n$. Therefore, its PSD is

$$\begin{aligned} S_{\mathrm{e}} &= S_{d-Wx} && (10)\\ &= S_s + S_{m_0} + S_{m_1} + |W(z)|^2 S_{m_1} + |1 - H(z)W(z)|^2 S_n. && (11) \end{aligned}$$

Therefore, noise reduction ratio is

$$\begin{aligned} R &= \frac{S_{m_0} + S_{m_1} + |W(z)|^2 S_{m_1} + |1 - H(z)W(z)|^2 S_n}{S_{m_0} + S_n} && (12)\\ &= \frac{R_{\mathrm{pri}} + R_{\mathrm{pri}} R_{\mathrm{ref}} + R_{\mathrm{ref}}}{(R_{\mathrm{ref}} + 1)(R_{\mathrm{pri}} + 1)}, && (13) \end{aligned}$$

where

$$R_{\mathrm{pri}} = \frac{S_{m_0}}{S_n}, \qquad (14)$$

$$R_{\mathrm{ref}} = \frac{S_{m_1}}{|H(z)|^2 S_n}. \qquad (15)$$

R_{ref} means S/N of the monitor (the microphone).

Therefore, even if the algorithm is perfect, we cannot subtract the noise completely when the noise in adaptive cancellation loop is large.