Some Essential Calculations Involving Phase Camera

Razib Obaid

July 28, 2010

This page includes some general calculation involving power and energy incident on the Phase Camera.In order to do so, we need to consider the electric fields of the EM waves incident on it.

In our setup, we have four different beams incident on the camera: the carrier from the main laser, the reference beam from the AOM, and the two sidebands from the EOM. Since I am doing a general calculation, I will assume that just before the reaching the camera, each of the beams have different phase shift. For a simplified calculation, please see my first project report.

Let the net electric field incident on the camera be E_{inc} .
So,

$$E_{inc} = A_c e^{i(w_c+\phi_c)t} + A_{sb} e^{i(w_c+w_{sb}+\phi_{sb})t} + A_{-sb} e^{i(w_c-w_{sb}+\phi_{-sb})t} + A_r e^{i(w_c+w_r+\phi_r)t} \tag{1}$$

where,

$$\begin{aligned}
A_c &= \text{Electric field of the carrier} \\
w_c &= \text{Frequency of the carrier} \\
\phi_c &= \text{Phase of the carrier} \\
A_{sb}, A_{-sb} &= \text{Electric field of the rhs sideband and lhs rideband} \\
w_{sb} &= \text{Frequency of the sideband} \\
\phi_{sb}, \phi_{-sb} &= \text{Phase of the rhs and the lhs sideband} \\
A_r &= \text{Electric field of the reference} \\
w_r &= \text{Frequency of the reference} \\
\phi_r &= \text{Phase of the reference}
\end{aligned}$$

Since $P = E^* \cdot E$, so

$$\begin{aligned}
P = |A_c|^2 &+ A_c A_{sb} e^{i(w_{sb}+\phi_{sb}-\phi_c)t} + A_c A_{-sb} e^{-i(w_{sb}+\phi_c-\phi_{-sb})t} \\
&+ A_c A_r e^{-i(-w_r+\phi_c-\phi_r)t} + A_c A_{sb} e^{-i(w_{sb}+\phi_{sb}-\phi_c)t} + |A_{sb}|^2 \\
&+ A_{sb} A_{-sb} e^{-i(2w_{sb}+\phi_{-sb}-\phi_{sb})t} + A_r A_{sb} e^{-i(-w_r+w_{sb}+\phi_{sb}-\phi_r)t} \\
&+ A_c A_{-sb} e^{i(w_{sb}+\phi_c-\phi_{-sb})t} + A_{sb} A_{-sb} e^{i(2w_{sb}+\phi_{-sb}-\phi_{sb})t} + |A_{-sb}|^2 \\
&+ A_r A_{-sb} e^{-i(-w_r-w_{sb}+\phi_{-sb}-\phi_r)t} + A_c A_r e^{i(-w_r+\phi_c-\phi_r)t} \\
&+ A_r A_{sb} e^{i(-w_r+w_{sb}+\phi_{sb}-\phi_r)t} + A_r A_{-sb} e^{i(-w_r-w_{sb}+\phi_{-sb}-\phi_r)t} + |A_r|^2
\end{aligned} \tag{2}$$

$$\begin{aligned}
= |A_c|^2 &+ |A_{sb}|^2 + |A_{-sb}|^2 + |A_r|^2 + 2A_c A_{sb} \cos(w_{sb} - \Delta\phi_{c,sb})t \\
&+ 2A_c A_{-sb} \cos(w_{sb} + \Delta\phi_{c,-sb})t + 2A_c A_r \cos(-w_r + \Delta\phi_{c,r})t \\
&+ 2A_{sb} A_{-sb} \cos(2w_{sb} + \Delta\phi_{sb,-sb})t + 2A_r A_{sb} \cos(-w_r + w_{sb} - \Delta\phi_{r,sb})t \\
&+ 2A_r A_{-sb} \cos(-w_r - w_{sb} - \Delta\phi_{r,-sb})t
\end{aligned} \tag{3}$$

Since the camera is exposed for a short duration to this power, if we integrate the power for that duration, i.e the exposure time, then

$$\begin{aligned}
\int_{t_0}^{t_0+\tau} P dt = &\left(|A_c|^2 + |A_{sb}|^2 + |A_{-sb}|^2 + |A_r|^2\right)\tau \\
&+ 2\frac{A_c A_{sb}}{\kappa_{c,sb}} \left\{\sin(\kappa_{c,sb})(t_0+\tau) - \sin(\kappa_{c,sb})t_0\right\} \\
&+ 2\frac{A_c A_{-sb}}{\kappa_{c,-sb}} \left\{\sin(\kappa_{c,-sb})(t_0+\tau) - \sin(\kappa_{c,-sb})t_0\right\} \\
&+ \cdots + 2\frac{A_r A_{sb}}{\kappa_{r,sb}} \left\{\sin(\kappa_{r,sb})(t_0+\tau) - \sin(\kappa_{r,sb})t_0\right\} \\
&+ 2\frac{A_r A_{-sb}}{\kappa_{r,-sb}} \left\{\sin(\kappa_{r,-sb})(t_0+\tau) - \sin(\kappa_{r,-sb})t_0\right\}
\end{aligned} \tag{4}$$

where $\kappa_{\square,\square}$ represents frequency and phase variable for corresponding box indices.

In our case, we are producing a beat signal with the reference beam and one of the sidebands. So it is justifiable to assume that $\tau \ll \frac{1}{w_r - w_{sb}}$ which leaves the integrated power (energy) as,

$$\begin{aligned}
P\tau \approx &\left(|A_c|^2 + |A_{sb}|^2 + |A_{-sb}|^2 + |A_r|^2\right)\tau \\
&+ 2\frac{A_r A_{sb}}{\kappa_{r,sb}} \left\{\sin(\kappa_{r,sb}\tau)\cos(\kappa_{r,sb}t_0)\right\}
\end{aligned} \tag{5}$$

where $\kappa_{r,sb} = w_r - w_s + \Delta\phi_{r,sb}$

The first four part in the above equation makes up the DC signal and the last part is the oscillatory signal due to the beat between the AOM and the EOM.